QUANTUM SHAPE KINEMATICS

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Abstract: Shape dynamics is a theory first proposed by Julian Barbour which states that physics happen uniquely in the reduced configuration space of a theory. So far, studies in the area have focused on gravitational systems. Here we first contemplate on the implications of this idea on quantum mechanics. We summarize the idea of shape dynamics and then give physical configurations of multi qubit systems. Our aim in the grand picture, is to initiate a research program translating classical shape dynamics to quantum realm.

Keywords: Shape Dynamics, Qubits, Relational Physics

Introduction

Mach's principle may refer to many ideas. Hermann Bondi and Joseph Samuel list eleven distinct versions of the Mach's principle that can be found in the literature (Samuel, 1997). We adopt the Julian Barbour's definition (Barbour, The Definition of Mach's Principle, 2010).

First of all there is the configuration space of a theory. By removing the gauge degree of freedoms we obtain the reduced configuration space of a theory where all degrees of freedom are physical. This space is called the *shape space*. The Mach's principle states that (Barbour, The Definition of Mach's Principle, 2010) a point and a direction or a tangent vector in shape space determine the evolution of the system uniquely.

According to classical shape dynamics, in the Newtonian N-body problem the physical configurations are obtained when the rotation and scale degrees of freedom are removed (Barbour, Shape Dynamics. An Introduction, 2011). Hence for one or two particles in an empty universe there is no degree of freedom. Non-trivial Shape dynamics apply to the cases of three or more particles. For an introduction to shape dynamics, reader may refer to (Barbour, Shape Dynamics. An Introduction, 2011) and (Mercati, 2014).

So far the studies on shape dynamics focused on gravitational (Henrique Gomes, 2015) and classical aspects such as the arrow of time (Julian Barbour, 2014). Beginning with this paper, we would like to initiate a research endeavor that investigates consequences of classical shape dynamics in quantum phenomena.

Shapes of Qubits

We discuss quantum shape kinematics of multi qubit systems beginning with the cases of a single and double qubit systems. In this section qubits do not occupy positions in spacetime. Hence they have only internal degrees of freedom.

Single qubit system
We consider there exists only one qubit in the Minkowski spacetime and nothing else. The quantum state of the particle can be written as:

$$|\psi\rangle = \alpha|\uparrow\rangle + \beta|\downarrow\rangle, \exists \alpha, \beta \in \mathbb{C}.$$

However by rotating the coordinates and multiplying with a complex number we can always map $|\psi\rangle \mapsto |\uparrow\rangle$. Therefore we conclude that for one qubit there is no physical degree of freedom apart from its mere existence.

Double qubit system
Here we suppose there are two qubits in the universe. The basis vectors are $|\uparrow\uparrow\rangle$, $|\downarrow\downarrow\rangle$, $|\uparrow\downarrow\rangle$ and $|\downarrow\uparrow\rangle$. Because all we have is the angles between spins, the first two correspond to parallel spin case, $|parallel\rangle$, and the last two correspond to anti-parallel spin case, $|anti-parallel\rangle$. We can always rotate the state of first qubit into $|\uparrow\rangle$.

Hence the direction of the first spin is used to fix a direction in space. The physical basis vectors are $|\uparrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle$.

Multiple qubit systems

In this part we suppose there are N qubits in the universe. The basis vectors of the system are $|a_1\rangle \otimes |a_2\rangle \otimes \cdots \otimes |a_N\rangle$ where $|a_i\rangle$ for $1 \le i \le N$ can be either $|\uparrow\rangle$ or $|\downarrow\rangle$. Whatever the value of $|a_1\rangle$ we can always rotate it to $|\uparrow\rangle$. Hence we reduce one degree of freedom. We call the first qubit as *the reference qubit*. N qubit system has the degrees of freedom of $N-1$ qubit system. This is true for all $N \ge 1$.

Possible Objections

Interactions

One may object to this classification with the counter example of interacting two qubits via the Hamiltonian $-\gamma\overrightarrow{\sigma_1} \odot \overrightarrow{\sigma_2}$. The eigenstates of the systems are $|\uparrow\uparrow\rangle, |\downarrow\downarrow\rangle$ and $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ with energy eigenvalue $-\gamma$, whereas the state $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ has energy 3γ. The seemingly paradoxical point is that the states $|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$ and $|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle$ have different energies though they should correspond to the same $|anti-parallel\rangle$ state vector.

We need to note that in order for two qubits to *interact*, we need to introduce additional structure. In quantum shape dynamics, time evolution should be reached by considering the whole state of the universe. It will be a holistic theory.

For simplicity suppose that the interaction between the qubits requires another qubit. All in all the system becomes a triple qubit system. Here the physical states are such that the additional qubit has always the state $|\uparrow\rangle$. Basis vectors are $|\uparrow\rangle \otimes |a\rangle \otimes |b\rangle$ where $|a\rangle$, $|b\rangle$ can be $|\uparrow\rangle$ or $|\downarrow\rangle$. It is now that the states $|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle$ and $|\uparrow\uparrow\downarrow\rangle + |\uparrow\downarrow\uparrow\rangle$ are physically different and there is no paradox for them having different energy values. The problem is solved once we take into account the whole system.

Choice of the reference qubit

The choice of the reference qubit, the qubit whose state is fixed, is arbitrary. By definition it has no dynamics, though it may evolve relative to subsystems.

Conclusion

In this study, we removed the rotation gauge degrees of freedoms from multi qubits systems and what have remained are the quantum shapes of the system. We observed that one qubit can be put in a fixed state by rotating the space, hence it has no degree of freedom. In the general case of an N qubit system, the degrees of freedom turned out to have the degrees of freedom of $N-1$ qubit system.

Future works may expand our discussion by including quantum shape kinematics of molecules or higher spin systems. Quantum shape dynamics solution for the Hydrogen atom would be an interesting example to see. It seems that there will be a difference on the formation of a single hydrogen atom in the universe compared to formation of $N \gg 1$ hydrogen atoms.

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