Integrating Writing Into Mathematics Clasroom As One Communication Factor

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ABSTRACT

Students in mathematics classrooms are expected to communicate not only using algebraic and geometric language, but also using other language modes to be able to share explicitly their mathematical thinking with others. Research has revealed that only high-achieving students are able to communicate by using algebraic and geometric representation. Recently, writing in mathematics classroom has received increased attention. The present study focuses on integrating writing into the mathematics classrooms by constituting a mathematical communication model. Data were obtained from 229,967 7th grade students who took the Texas Assessment of Knowledge and Skills high-stakes test in the areas of mathematics and writing. A second-order Confirmatory Factor Analysis was used to create mathematical communication model. The fit indices showed that the model was a good fit for the data.

Keywords: writing in mathematics, communication in mathematics, mathematical communication.

INTRODUCTION

National Council of Teachers of Mathematics (NCTM) emphasized the importance of communication by placing communication into two NCTM's calls in 1989 and 2000. These two calls conveyed the idea that communication is an integral part of mathematics classrooms, and it is crucial for students to clarify and develop their mathematical thinking and understanding (NCTM, 1989; 2000). Through a language, students in mathematics classrooms transmit their mathematical thoughts to others; thus enabling them to construct a model of their mathematical thinking (Sierpinska, 1998). Neria and Amit (2004) conducted a study to determine students' preferences across different communication modes in mathematics, and found that the only high achieving students' preferred algebraic communication to represent their mathematical solutions. Cai et al. (1996) reported that students' preference in representing their mathematical solutions was verbal rather than algebraic and geometric modes. Later, Nathan and Koedinger (2000) revealed that students had preferences in explaining their reasoning in non-algebraic modes because algebraic modes were too abstract; thus making students' communication difficult (Hembree, 1992). To enable not only high achieving students but all students to communicate effectively, verbal modes of communication need to be integrated into mathematics classrooms.

Using writing in mathematics classrooms has received increased attention. Seto and Meel (2006) highlighted the importance of writing by noting one of the crucial changes over the past couple decades in mathematics teaching and learning was using writing as a communication tool in mathematics classroom. Written communication in mathematics classrooms is important because students write using communication tools that reflect their mathematical understanding, and involves the mathematical community (Fried & Amid, 2003; Morgan, 1994).

Therefore, written communication in mathematical instruction needs specific attention to help students become more familiar and comfortable with mathematical vocabulary, phrases, shapes, and meanings (Thompson & Rubenstein, 2000). The aim of the present study was to determine how written communication along with algebraic and geometric communication modes constitutes a communication model in mathematics classrooms.

THEORETICAL FRAMEWORK

Communication in Mathematics Classrooms

As stated in NCTM (2000), all students in mathematics classrooms are expected to communicate both by using algebraic and geometric language, and by using other communication tools to share explicitly their mathematical thinking with others. The more students develop mathematical communication, the deeper their mathematical thinking and reasoning skills (NCTM, 1989; 2000). Capraro, Capraro, and Rupley (2011) recently noted that while mathematics is itself a language (i.e., algebraic language, and geometric language) for communication, there are some other useful communication tools that can be employed in mathematics classrooms to increase students' mathematical understanding.

A language, in its most general spectrum includes four main communication components: reading, writing, listening, and speaking. In traditional mathematics classrooms, the most commonly used communication tool is listening because students in classrooms spend most of their time listening to direct lectures without having an opportunity to use the other three communication tools. However, as in the Curriculum and Evaluations Standards (NCTM, 1989) stated,

The development of a student's power to use mathematics involves learning the signs, symbols, and terms of mathematics. This is best accomplished in problem situations in which students have an opportunity to read, write, and discuss ideas in which the use of the language of mathematics becomes natural. As students communicate their ideas, they learn to clarify, refine, and consolidate their thinking (p. 6).

Burley-Allen (1982) noted that students in K-12 do not lack training in written and reading communication, but it is still unclear whether within reading and writing instructional time students learn how to communicate with mathematical text. Such mathematical texts may require students to have mastery language knowledge in at least reading and writing to capture the messages of texts. In order for students to overcome mathematical language challenges, teachers should integrate various modes of communication into mathematics classrooms, and they should avoid using the words, phrases, and concepts that are familiar to teachers but foreign to their students (Thompson & Rubenstein, 2000). The language of mathematics is limited to school, and most K-12 students do not hear, see, and use the mathematical words, phrases, and concepts in their daily-lives. Because students do not see the application of mathematical words, concepts, and terms, they cannot reach a deep and personal understanding of mathematical facts and algorithms, and their learning become more rote memorization rather than meaningful learning. To make students' mathematical learning more meaningful, teachers should allow students to communicate with their existing knowledge and experiences before introducing brand new mathematical words, terms, and concepts. Therefore, "we need to be sensitive to many issues related to the language of mathematics and students' growing fluency with it" (Thomson & Rubenstein, 2000, p. 1)

Thompson and Rubenstein (2000) noted that the language of mathematics play an essential role at least in three following perspectives: 1) teachers use language to teach as their major means of communication. Adam (2010) emphasized the importance of language in mathematics classroom by noting that no mathematics would be if no language were, 2) students construct, and develop mathematical ideas by using language as a communication tool. According to Skemp (1976), there are two levels of mathematical language: a) surface level of mathematical language, and b) deep level of mathematical language. Students in the surface level need to construct enough mathematical vocabulary to be able to pass to deep level of mathematical language that they can discuss mathematical ideas. 3) Teachers assess students' mathematical understanding through language. NCTM (1989) emphasized that asking questions to understand students' prior mathematical experiences is one of the crucial parts of mathematics instruction in K-12. Teachers need language to be able to assess students' mathematical understanding. Unless teachers ask what students' thinking, they cannot understand how their ideas (Ball, 1994) need to be changed or developed. For example, through reading students' writing, teachers can understand their students' mathematical thinking, reasoning, and understanding; thus they can determine students' weakness and strengths (Bell &Purdy, 1985) in mathematics. Crespo (2000) suggested that teachers can use written and oral communication to assess students' mathematical understanding; however, Ashlock (2006) noted that using written communication in mathematics classroom to assess students' mathematical understanding are more reflective than using oral communication. Later, Bicer, Capraro, and Capraro (2013) added that integrating writing into mathematics classroom is helpful communication practice for teachers to understand their students' mathematical ideas, feelings, and beliefs towards mathematics.

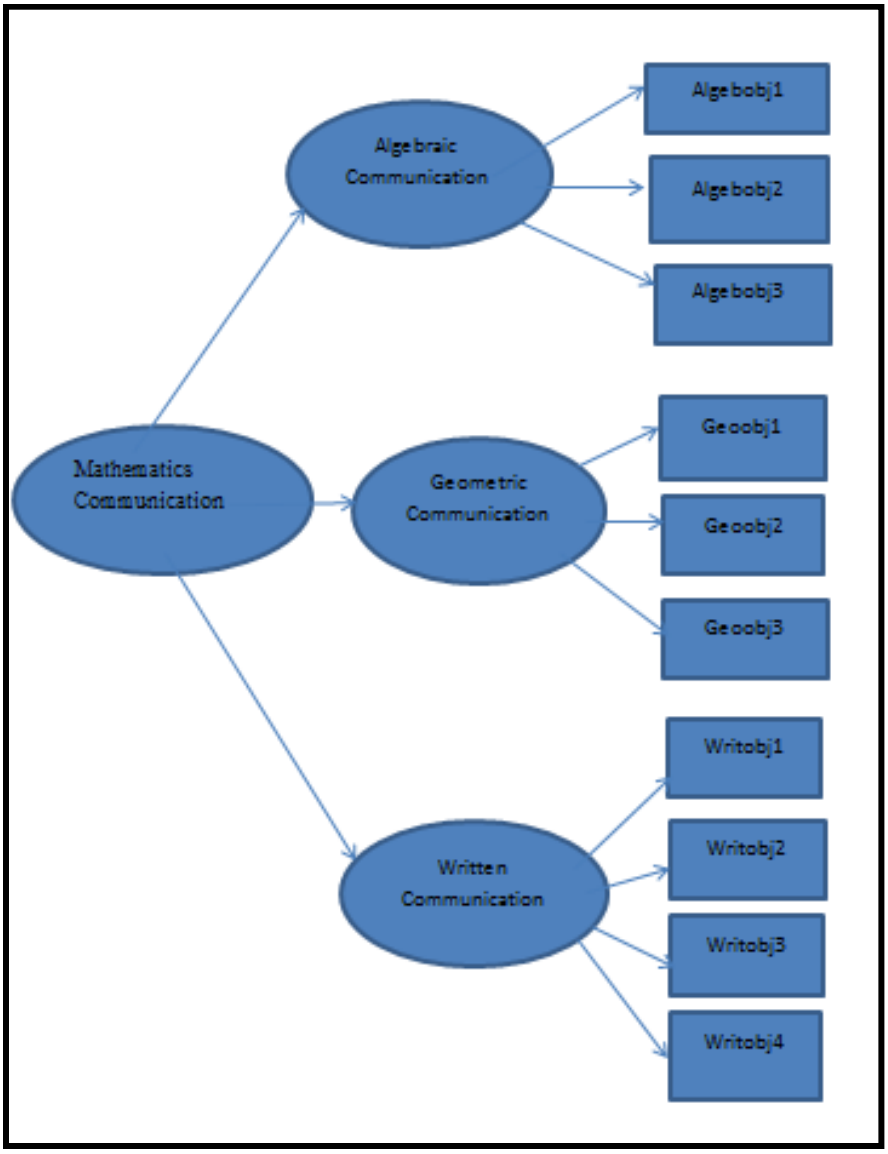
Communication in mathematics is directly and strongly related to problem solving and posing. In order to be a good problem solver in mathematics, students should have two skills: 1) problem representation skills which include words, graphs, pictures, and tables, and 2) symbol manipulation skills which include being able to carry out mathematical and geometrical procedures (Brenner et al., 1997). Students' mathematical problem solving skills mostly depend on their representational skills of mathematical thinking (Brenner et al., 1997). This is because representations enable student to externally reflect on their problem solution processes (Cai, Magone, Wang, & Lane, 1996). Wheeler (1996) noted that students in mathematics classroom use many tools including but not limited to tables, graphs, formulas, equations, arrays, identities, and functions. In order for learners to effectively use these tools, they "must first understand the associated representations and how to link them together" (Drisccoll, 1999, p. 141). Using representational techniques to increase students' problem solving skills can take the form of various oral and written actions; however, the main purpose of using various communication tools is to enhance students' mathematical reasoning (Pugalee, 2001). Borasi et al. (1998) and Gardner (1983) noted that students think and process problems in many ways including written, visual, kinesthetic, and oral forms. Students in mathematics classroom should have options for representing their problem solution processes symbolically, verbally, and graphically or diagrammatically (Shield & Galbraith, 1998). In addition, students should be flexible in translating from one representation to another. Driscoll (1999) noted that students who are flexible in using multiple representations have better mathematical perspectives than students who are using only one representation mode and are not flexible in translating from one to another representation mode. Davis (1987) noted that students who are able to translate tables, equations, and graphs for functions have the potential for understanding various vital connections between algebra and geometry. NCTM has called for the use of multiple representational modes in mathematics classroom by suggesting that students should use tables, graphs, pictures, and words before they begin using formal mathematical language like equations, formulas, and functions.

Writing as One Communication Factor in Mathematics Classroom Using writing as one communication tool in mathematics classrooms has received increased attention (Meiner & Rishel, 1998). The reason why writing has received so much attention in mathematics classroom is because writing develops students' mathematical content learning (Meel, 1999), and students' problem solving skills (Bagley & Gallenberger, 1992). Integrating writing into mathematics classroom as one mode of communication and representation develops students' problem solving skills (NCTM, 2000) because it requires students to reflect on their reasoning during the problem solution process (Banger-Drowns, Hurley, & Wilkinson, 2004). Through writing, students gather, analyze, and interpret mathematical questions; thus enabling them to externalize internal representations for directs interpretation (Nahrgang & Petersen, 1986). Duke and Pearson (2002) and Haneda and Wells (2000) found that students in mathematics classrooms with integrated writing had deeper mathematical comprehension than students in mathematics classroom without writing, and Kreeft (1984) found that students' metacognition develops as they become aware of and control their mathematical thinking and understanding. Writing in mathematical classroom is helpful practice because it provides students an opportunity to represent their mathematical reasoning such as; analyze data, compare and contrast mathematical problems, and synthetize relevant mathematical knowledge (Emig, 1977). Bicer, Capraro, and Capraro (2013) noted that mathematical questions sometimes can be complex or difficult for some students because they have difficulty organizing their thoughts by using either algebraic or geometric language. However, writing can help students in organizing their mathematical thinking; thus becoming problem solvers even when they face difficult or complex mathematical questions. "Thus, the writing process may encourage students to solve difficult problems because writing makes difficult problems more concrete rather than an abstract or imaginary thing" (Bicer, Capraro, & Capraro, 2013, p. 366). Because writing requires students to represent their mathematical imagination, their mathematical thinking becomes more concrete, original and insightful (Nagin, 2003). To summarize, writing in mathematics classroom fosters students reasoning skill (Swafford & Bryan, 2000) by converting more complex mathematical questions into concrete ones, and develops their metacognition (Kreeft, 1984; Stanton, 1984) by providing opportunities for them to see what and how they know mathematical terms, axioms, or theorems (Bicer, Capraro, & Capraro, 2013).

Hypothesized Model

Research in mathematics education has emphasized the importance of communication including algebraic, geometric, and written; however, there is no existing model in the literature to represent the component of mathematical communication together. Therefore, in the present study, algebraic, geometric, and written communication modes are considered as mathematical communication modes and these together constitute a hierarchical (higher order) model. The present study seeks to answer the following question: Does the model (integrating written communication along with geometric and algebraic communication into mathematics classroom) yield a fit model to the data?

Figure 1: Hypothesized model with observed variables.



METHODS

In the present study, structural equation modeling (SEM), which is a frequently used statistical approach in education (Thompson, 1998), is employed to determine if the three communication modes in mathematics classroom namely algebraic, geometric, and written together constitute a higher order model. We consider written communication as one factor in addition to algebraic and geometric factors, and these three factors together constitute a hierarchical communication model. Higher-order confirmatory factor analysis is employed because it has the capacity to test complex hierarchical dynamics of the model (Thompson, 2006).

Data Sources

Data were obtained from the Texas Education Agency, the administrators of the Texas Assessment of Knowledge and Skills (TAKS) test every year. For the present study, data were gathered from 229,967 students (48% of male & 52% of female) who were 7th graders, and (high stakes test in) took the TAKS test in 2011. Only scores measuring the mathematics and writing objectives (see in Table 1) of students were taken into consideration.

Table 1: TAKS Mathematics and Writing Objectives

TAKS at Grade 7	Objectives	Expectations from students
TAKS Mathematics	Objective 1	Understanding of numbers. Operations, and quantitative reasoning
	Objective 2	Understanding of patterns, relationships, and algebraic reasoning
	Objective 3	Understanding of geometry and spatial reasoning
	Objective 4	Understanding of the concepts and uses of measurement
	Objective 5	Understanding of probability and statistics
	Objective 6	Understanding of the mathematical process and tools used in problem solving
TAKS Writing	Objective 1	Produce and effective composition for a specific purpose
	Objective 2	Produce a piece of writing that demonstrates of the conventions of spelling, capitalization, punctuation, grammar, usage, and sentence structure
	Objective 3	Recognize appropriate organization of ideas in written text
	Objective 4	Recognize correct and effective sentence construction in written text

The mathematics objectives were divided into two parts: algebraic communication and geometric communication. Observed variables on algebraic communication were chosen as TAKS mathematics objectives 1, 2, and 5. The selection of these objectives for algebraic communication was based on representational technique requiring students to represent their solution with an equation, function, and arithmetic computation. Observed variables for geometric communication were chosen as TAKS mathematics objectives 3, 4, and 6. This selection was based on representational technique requiring students to represent their mathematical solution with either diagrams, graphics, and /or pictorial illustrations. For writing, objectives 1, 2, 3 and 4 selected, as observed variables and the selection was based on representational technique requiring students to represent their justification with words. The hypothesized model with latent and observed variables is illustrated in Figure 2. Mplus was employed to determine how the hypothesized model fit the data set. Missing data is dealt with the default in Mplus.

Evaluation of Fit Model

The values of the model as follow: a) chi-square= 10935 (p < 0.01); b) degrees of freedom= 34; c) comparative fit index (CFI) = .993; d) root mean square error of approximation (RMSEA) = .037; e) standardized root mean square residual (SRMR) = .022.

Because chi-square tests are sensitive to sample size, other fit indices need to be considered. Due to large sample size, the chi-square yielded a large quantity, but it was still statistically significant. The other fit indices (RMSEA, CFI, and SRMR) showed that the model was a good fit for the data. To obtain a good model fit, it is suggested that the RMSEA index SHOULD suggested to be lower than .06, CFI index higher than .95 (Hu & Bentler, 1999), and SRMR index lower than .05. According to the model, each path between variables was statistically significant. In Figure 3, the standardized estimates were shown. In Table 2, R-square values for each variable were larger than .5 (p < .05) and thus practically important.

Figure 2: Standardized parameter values of the model.

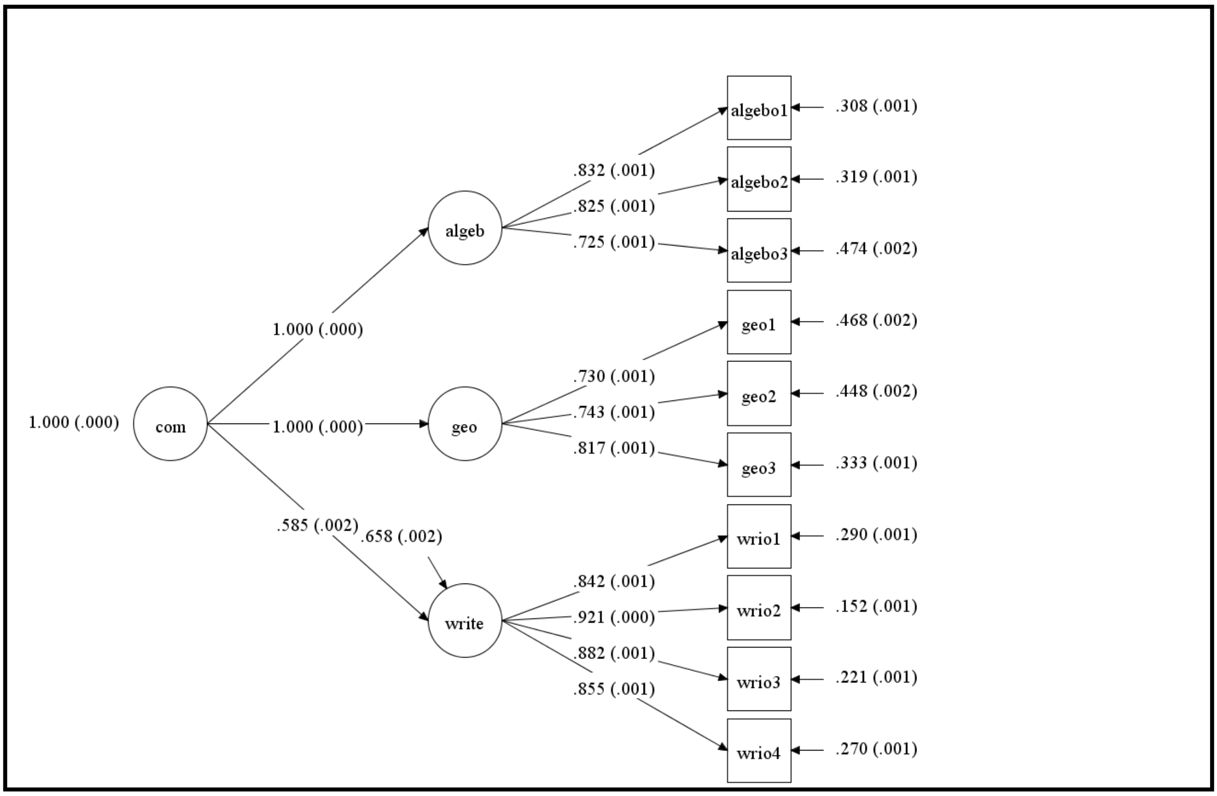


Table 2: R-squared and p Values of Observed Variables

Observed Variable	***R-square***	***p-value***
Algebo1	.692	< .001
Algebo2	.681	< .001
Geo1	.532	< .001
Geo2	.552	< .001
Algebo3	.526	< .001
Geo3	.667	< .001
Wrio1	.710	< .001
Wrio2	.848	< .001

Wrio3	.779	< .001
Wrio4	.730	< .001

DISCUSSION

Writing as one of the communication modes emphasizes the development and deepening of mathematical thinking and reasoning (Neria & Amit, 2004). The present study adds writing as one communication mode into mathematics classrooms along with algebraic and geometric communication to constitute a higher order communication model. The result suggested that these three communication modes together constitute a hierarchical communication in mathematics, and the model yields fit to the data.

Recent research emphasized that integrating writing into mathematics classroom could increase students' problem solving skills (Bicer, Capraro, & Capraro, 2013), but there has been no mathematical communication model which includes various communication modes, and how these communication modes works together in promoting mathematical instruction. The reason why mathematics instruction needs a communication model which includes various communication modes is because research in the mathematics education era has revealed that many students are not flexible in communicating by using algebraic and geometric representation tools to clearly communicate with mathematical problems (Neria & Amit, 2004; Cai et al., 1996, Nathan & Koedinger, 2000). Cai et al. (1996) found that students' preferences were to communicate their mathematical solution in a verbal mode rather than non-verbal (algebraic and geometric modes) (Cai et al., 1996). The reasons why writing needs to be integrated into mathematics classroom along with algebraic and geometric modes of communication are noted in the following paragraphs.

First reason is that writing helps students organize their mathematical thoughts (Bagley & Gallenberger, 1992). Bicer, Capraro, and Capraro (2013) stated, "Some students are not flexible in organizing their thoughts about problems due to either the complexity or difficulty of the problems" (p. 366). However, various writing activities in mathematics classroom can assist students in overcoming the complexity of problems. For example, teachers may use a template outlining general problem solution steps; thus students who have difficulty organizing their mathematical thoughts can follow the template, solve the problem, and more comfortably go back to check their solution. In addition teachers, parents can use writing as one communication tool to assist their children mathematics assignments. Using writing as one communication mode in mathematics may be easier for less-educated parents compared to algebraic and geometric communication modes. Bicer, Capraro, and Capraro (2013a) noted the importance of parental communication on children's mathematics achievement; thus integrating writing in mathematics can be one way to increase parent-child communication. This is important because Bicer, Capraro, and Capraro (2013b) suggested that parents should be able to communicate with their children mathematical understanding and know the mathematical requirements they exposed; thus enabling parents to solve their children mathematical problems by using written communication mode once algebraic and geometric communication modes are too abstract for them.

The second reason why writing as a communication mode needs to be considered in mathematics classroom is because algebraic and geometric communication modes are too difficult and abstract for some students to use when they need to share their mathematical result with others (Neria & Amit, 2004). To illustrate, Hembree (1992) noted that using numbers which were generalized by symbols or letters is an abstraction that makes students' mathematical knowledge construction imaginary. Communicating with previously learnt symbols to learn new mathematical ideas sometimes creates an obstacle for students going forward in mathematics. Herscovich and Linchevski (1994) and Lee and Wheeler (1989) suggested that students should first construct a solid foundation by using numbers and words, but later they can use more abstract modes of mathematical communication (e.g. algebraic communication) to solve mathematical problems. However, the mathematical vocabulary students learn and use during instruction to solve mathematical problems is mostly formal and students do not have an opportunity to see the relationship between the informal (everyday language) and formal mathematical words (Crillo, Bruna, & Herbal-Eisemann, 2010); thus their mathematics learning become less meaningful. Teachers should allow students to use everyday language in their mathematical writing to make their learning more meaningful. When teachers in mathematics classroom encourage students to write down their solutions by using everyday language words, they can later see the connections among the mathematical terms, symbols, definitions, and axioms. Hence, their mathematical learning may be more concrete rather than an imaginary thing.

The third reason is mostly related to geometry rather than algebra. In geometry classrooms, students should develop their mathematical imagination in order to be able to solve geometrical problems. Bicer, Capraro, and Capraro (2013) noted that one reason why students have difficulty interpreting geometrical problems is due to their lack of spatial thinking or mathematical imagination. Van Hiele (1973) reported a model for geometry teaching and learning that includes five essential levels; 1) visualization, 2) analyzing, 3) generalization, 4) deduction, and 5) rigor. In this model, students who have difficulty visualizing geometrical problems are not expected to pass to the next level because the model developed by Van Hiele (1973) was constructed in hierarchical order. Before students attain the highest level of geometrical thinking (rigor), they need concrete examples and proofs to be able to solve and understand geometrical problems requiring higher level of geometrical thinking. Students at the highest level start thinking mostly in a more geometrical abstract manner. To make students transition to each level easily and enable them to achieve the highest level, teachers can integrate writing into geometry classrooms to make geometrical questions more concrete for students who need to be supported to extend their geometrical imagination. Once students have difficulty with visualization of geometrical shapes (e.g, hexagon, polygon, cubes, and ellipse), they can sketch pictures, figures, or graphs to make the abstract geometrical questions more concrete (Bicer, Capraro, & Capraro, 2013). Writing may be a helpful practice to enable students to achieve the highest level of geometrical thinking; thus students may be able to think in a more abstract manner without having concrete examples.

The last reason why writing needs to be integrated into mathematics classroom is because it not only helps students to develop their mathematical reasoning, metacognition, and higher levels of thinking, but also helps teachers assess students' mathematical understanding. When teachers allow students to write down their mathematical thinking, understanding, or solution, they become owners of their learning (Mayer, Lester, & Pradl, 1983); thus mathematic classrooms become more student center rather than teacher centered one. Writing also helps teachers diagnose students' mathematical misconceptions, strengths, weakness, and feelings towards certain mathematical content. The reason why writing is emphasized is to diagnose allowing teachers to read each student's response carefully. This cannot give in the same time period and with the same effort when a teacher has to listen to each student's oral explanation. To have each students have a voice writing may be the best communication tool enabling teachers to hear and understand what students know and how their mathematical ideas need to be developed or changed.

Overall, writing is one communication mode that provides potential benefits for mathematics teaching and learning; thus needs to be integrated into mathematics classrooms. As stated above, writing enables not only students to increase their mathematical reasoning and representation, but also makes students aware of their mathematical understanding (Kreeft, 1984; Nagin, 2003; Swafford & Bryan, 2000). Writing also helps teachers to assess each student's mathematical understanding in an appropriate way. "Rather than just scoring papers, we need to understand each student's paper diagnostically-looking for patterns, hypothesizing possible causes, and verifying our ideas" (Ashlack, 2006, p. 15). The present study confirmed previous experimental research findings that writing is one factor that can be integrated into mathematics classrooms, and written communication along with algebraic and geometric communication constitute a hierarchical mathematical communication model. As one limitation of the present study, there is no oral communication added into mathematical classroom communication model. Future studies should explore the second order model by adding oral communication observed into mathematic classroom communication.

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