THE PLACE OF PROBLEM SOLVING AND MATHEMATICAL THINKING IN THE MATHEMATICAL TEACHING

Esen ERSOY[1], Pınar GÜNER[2]
[1,2]Ondokuz Mayıs University, Faculty of Education, Samsun, 55160, Turkey
[1]www.esene@omu.edu.tr, [2]www.pinar.guner@marmara.edu.tr

ABSTRACT

The purpose of this study is to investigate the effect of problem-solving subject given for the improvement of problem-solving skills and for teaching the problem-solving strategies on primary preservice mathematics teachers' ability to use the problem-solving stages and on their mathematical thinking levels. The study is oriented to descriptive survey, which is one of the quantitative research methods. In the study that was conducted throughout 13 weeks (26 hours), the students were taught Polya's (1945) problem-solving stages, which are formed of four steps, and they were also the problem-solving strategies in order to improve their problem-solving skills. In the study, two problems developed by Posamentier and Krulik (1998) was used as data collection tool, and "Mathematical Thinking Scale" developed by Ersoy (2012) was used to determine whether the problem-solving lesson has an effect on mathematical thinking. The findings obtained demonstrate that problem-solving subject has a positive effect on preservice mathematics teachers' improving their problem-solving skills, and their skills of being able to choose and practise proper strategy. In addition, it was concluded that problem-solving subject has a positive effect on mathematical thinking.

Key Words: Mathematical thinking, problem solving, problem-solving stages.

INTRODUCTION

Problem and problem solving is the indispensible fact of life and an integral part of mathematics. Problem is an effort which arouses the need to solve in the individual, whose way of solution the individual does not know and which the individual tries to solve (Charles and Lester, cited by Baykul 1999). While Polya (1945) defines problem as seeking for the appropriate step to be taken in a conscious way so as to eliminate the uncertainty but failing to reach the desired one, Dewey describes as a situation leading to uncertainty confusing human intellect (Baykul 1999). According to Baki (2006), problem is a notion which arouses uneasiness in the individual and therefore leads him to seek for a solution using his own knowledge and experience. The above-mentioned definitions presents the three main characteristics of problem: it is difficult for the person who encounters it, it arouses the need to solve in the individual and the person encounters it first time at that moment and thus the individual has no preparation pertaining to its solution (Altun 2005). According to Dewey, if a problem is situation which creates uncertainty in the human intellect, then problem solving is the elimination of these uncertainties (Tanrıseven 2000). In a broader sense, problem solving is cognitive and affective process that covers devising various alternative ways and then selecting and applying the suitable one among those ways in order to cope with the uncertainty (Güçlü 2003). Solving a mathematical problem is, on the other hand, the reflection of the processes which advocate to reach a solution and to make generalization (Sağlamer 1980). Polya (1945) defines the process of problem solving in four steps: a) understanding the problem, b) determining the strategy, c) applying the selected strategy, and d) assessment.

In the stage of understanding the problem, the student is expected to understand what the problem means, to determine what are the given and what are asked and to clearly present the condition of the problem. In the stage of determining the strategy, the student is expected to determine which steps of calculations, drawings, etc. to follow in order to reach the desired outcome. In this process, teacher can write all of the strategies on the board and can promote the use of different problem-solving strategies, and can enable them to choose the suitable strategies (Miller 2000). The following stage covers the implementation of the established plan by the student. Teacher must follow the student in each stage. In the assessment stage, the student is expected to control whether his solution is correct and meaningful. Turning back to the solution and considering it again make it possible for the knowledge to become strong and for the problem-solving skills to be improved. In literature, problem-solving stage were divided by different researches into different stages (Bransford and Stein 1993; Charles, Lester, and O'Daffe 1994; Dewey 1995; Sağlamer 1980). However, in this study, Polya's four problem solving strategies were used.

One of the important components of the problem-solving process is, on the other hand, the problem-solving strategies. The use of suitable problem solving strategy is significant in terms of being successful in problem solving. Problem solving strategy is a plan made as to how a question can be solved, and a perspective and pattern in the events (Mintzberg 1994). The most common problem solving strategies included in the literature are guessing and testing, making a systematic list, finding a pattern, drawing a diagram, solving a equation inequation, estimating, solving a simplier problems, working backwards, making a table and logical reasoning. While Hatfield, Edwards and Bitter (1997) state that in the selection of problem solving strategies, individual's experience and the mathematical knowledge he possesses are effective, Willoughby (1991) advocates the idea that problem solving steps and teaching its strategies in advance will improve the individual's problem solving skills (Israel 2003). Burkhardt (1994) supports the view that the teaching of problem solving process is mathematically, pedagogically and individually difficult in terms of teachers. Therefore, it is thought that the use of appropriate problem solving steps and strategies included in the literature will ease the teachers' job in the teaching of problem solving.

NCTM (2000) advocates that all of the students, starting from the pre-school, should be made to acquire the behavior of building mathematical knowledge, being able to solve problem not only in mathematics but in every field as well, applying the proper problem solving strategies, and evaluating the problem solving by means of problem solving. Because problem solving makes it possible to structure knowledge and to bring into connection with the other knowledge, it is included in the center of mathematics programs. The targets in the programs are generally to raise individuals who can cope with various problems and to improve the problem solving skill (Şahin 2007).

In the eyes of mathematicians, mathematics is the single method of thinking that leads us to certain knowledge (Yıldırım 2004). It is required that the problem be determined for thinking to develop. The individual tries to solve the problem by correlating among the concepts for the solution of problem, and at this stage, thinking starts (Ersoy 2012). Individual's mathematical thinking skill improves in the problem solving stage.

Practicing on mathematics problems makes it possible for strategies to be developed aimed at the rational solutions of problems and enables these strategies to be adapted to all kinds of problems to encountered in life by leading to mathematical thinking (Yavuz 2006). The objective of mathematics teaching is, on the one hand, to enable individual to solve problem and to teach problem solving stages, on the other, to allow him to think mathematically. In mathematics teaching, individual's transition to thinking process should be ensured in the problem solving stages.

Students' mathematical thoughts in problem solving and creating a problem should be formed by starting out the figures in the sample. In order to solve problems, students reach the results selecting the appropriate solving strategies and making contact with each other at the solving stage (Cai 2003). The fact that the teachers who provide education to students aiming at solving problem which is an important part of mathematics education should be well-supported is significant in terms of raising individuals to be able to solve their problems in every sense. Because, students' acquisitions and skills as to problem solving will take form depending on the teacher's approach of problem solving and level of knowledge that he possesses. Therefore, this research focuses on the pre-service teachers' (who will be teachers in the future) skills of understanding the problem, determining and using appropriate strategies and evaluation, and their levels of mathematical thinking.

Problem Status

1- Does the subject of problem solving have an effect on the students' skills of understanding the problem, determining and applying the appropriate strategy for it and evaluating the strategy?

2- Does the subject of problem solving have an effect on the students' mathematical thinking levels?

METHOD

The research is oriented to descriptive survey, which is one of the quantitative research methods. According to Aggrwal (2008), it is a research method which gathers information about current conditions or situation in order to analyse, describe, interpret and compare the trends and relationships (Salaria 2012). The data obtained during the study, doing the statistical assessments and reaching a conclusion concerning the population are fundamental in quantitative research (Ersoy 2013).

In this study, several practices were made in the subject of "Problem Solving in Mathematics", which is one of the third year subjects of primary mathematics teaching, in order to improve the problem solving strategies. It was ensured that the students should analyze the problem solving phases considering Polya's (1945) problem solving stages. In the problem solving stage, the students were assessed in terms of the stages of understanding the problem, being able to select and apply the involved strategy, and being able to evaluate it. In addition, an attempt was made to determine in which way the subject of "Problem Solving in Mathematics" affects the mathematical thinking skills.

In the research, process of problem solving and its teaching was taught as a subject during the 13 weeks (26 hours). At the beginning of the term, problem solving strategies were taught to the students for two hours. After that, two problems and mathematical thinking scales were applied. At the end of the 13th week, the same problems and scale were applied again. In the course of the term, various problems were solved and some practices were made.

In the analysis of the data obtained, we tried to determine whether the students employed the problem solving stages in the solutions of the problems. In the problem solving stages, we examined the processes of understanding the problem, being able to select and apply the involved strategy, and being able to evaluate it. In the study, we attempted to determine whether the students solved the given problem using problem-solving stages and to specify their mathematical thinking levels.

Study Group

The study group was formed of randomly chosen 40 students, who take the subject of "Problem Solving in Mathematics", which is one of the third year subjects, in the Department of Mathematics Teaching in Primary Education in the Faculty of Education at a public University in Samsun, in 2013-2014 academic year. The number of students participating in the application at the beginning of the term is 23. The number of students attending the practice at the end of the term is 17.

At the stage of applying the problems and scale, all of the students taking part in the practice were ensured to answer the scales, but the students who imperfectly answered the problems and the scale were excluded from the assessment while the statistical analyses were being performed. Therefore, the number of samples differs from one another.

Data Collection Tools

Two problems, which were produced by Posamentier and Krulik (1998) and which were translated into Turkish, and "Mathematical Thinking Scale" developed by Ersoy (2012) was used as data collection tool in the study. The data collectin tools were explained below at full length.

The Problems

The problems included in the study were taken from the book named "Problem-Solving Strategies For Efficient And Elegant Solutions: A Resource For The Mathematics Teacher", which belongs to Posamentier and Krulik (1998), and was translated into Turkish. These problems are included below.

Problem 1. "There are 5 people in the same room, and what is the probability of at least two of their birthdays being same?

Problem 2. There are two peg tops below. We have two numbers at hand derived from (a) and (b). What is the probability of the sum of two numbers' being even?

Mathematical Thinking Scale

At the stage of developing the scale intended for mathematical thinking, the stages of developing scale suggested below were applied (Ersoy 2012).

The Stage of Item Pool: The questions in the created item pool were arranged on high level thinking, mathematical thinking, cognitive dimension, thinking level and individual thinking skill. Answer formats were specified on the interview form to be submitted to the expert opinion. An item pool of 32 pieces, which were formed of the first states of the items, was created. The positive items in the scale were scored from 5 to one in the ways of "I totally agree = 5", "I partially agree = 4", "indecisive = 3", "I don't agree = 2", and "I don't agree at all = 1", on the other hand, the negative expressions were scored from 1 to 5 in the ways of "I totally agree = 1" and "I don't agree at all = 5".

The Stage of Testing the Content Validity: The comparisons were evaluated in the forms that were submitted to the expert opinion. It became clear after the experts opinions that content validity of the scale is suitable.

The Stage of Factor Analysis (Construct Validity): In order to identify the factor constructs of the scale, the untransformed factor analysis method was employed at first using principal components analysis, and then Varimax vertical rotation method was employed according to principal components. The fact that the KMO value proved to be 0,759 according to the test conducted revealed that there were sufficient data in the research and that the sample size was appropriate. In addition, Bartlett test results were evaluated in order to point out that the data were applicable to the factor analysis. In consequence of the Bartlett test result (p=0,000), it became clear that the data were applicable to the factor analysis.

At first, factor analysis was carried out without using rotation. Most of the variables were included in the first factor. In the other factors, rotation was made using Varimax method because they could not explain the variables sufficiently. Therefore, the Varimax vertical rotation method was used for the items the load points of which were below 0,30 in the

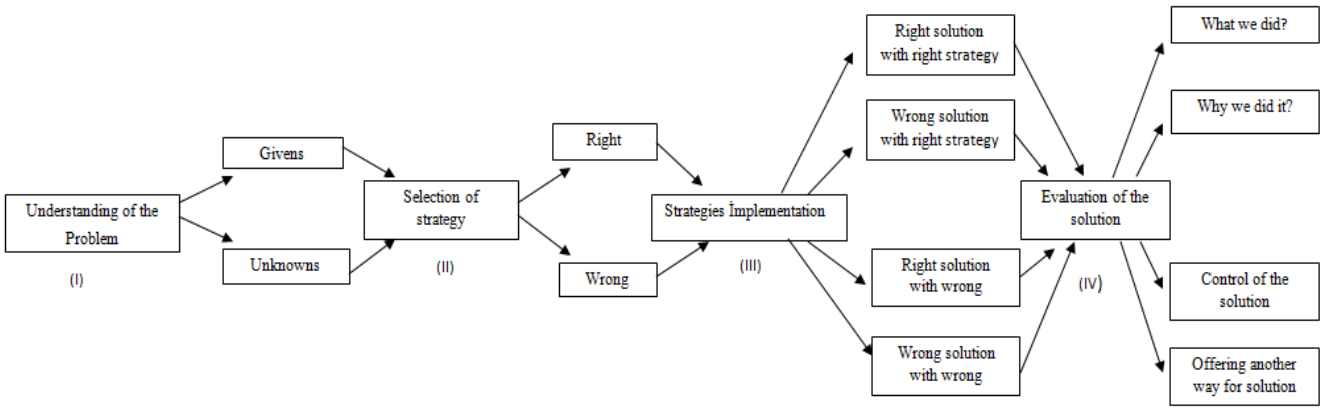
Component matrix. At this stage, the 8th, 10th, 11th, 18th, 25th, 26th, and 32nd items were excluded from the scale after having made some tests. It is observed that as the result of the performed analysis of the items whose factor loads were calculated, the scale was made up of 25 items and 4 dimensions. Explanatoriness was divided into 4 factors. By taking advantage of alternate factor loads, the sub-dimensions derived considering the meaning the items in the factors bears were encoded respectively as the tendency of high level thinking, reasoning, mathematical thinking skill and problem solving.

Stage of Reliability: In consequence of the analysis performed, reliability of the scale as calculated as 0,78. The conclusion drawn proves that "Mathematical Thinking Scale" is valid and reliable. As a result of the data obtained, the mathematical thinking scale was formed of 25, including 20 positive and 5 negative, items. The negative items included in the scale were encoded reversely. The highest score to be got is 125, and the lowest one is 25. It becomes clear that as the scores got from the scale increase, the mathematical thinking level increase, but as the scores got from the scale decrease, the mathematical thinking level decreases. In the practice of the scale, the students were given 15 minutes time.

Collection of the Data

- The data were collected in the subject of "Problem Solving in Mathematics" at the first semester of 2013-2014 academic year.
- Mathematical Thinking Scale was applied to all students at the beginning of the term.
- The application period of the subject was completed in the 13 weeks.
- The students were taught the problem solving stages for a hour at the first week of the course.
- Polya's (1945) problem solving stages were taken into consideration at the problem solving stages.
- In the second week of the course, two problems developed by Posamentier and Krulik (1998).
- Various problems were solved in the class till the end of the term and it was discussed what kind of strategies these problems could be solved with.
- During the term, the students formed triple groups. They solved some problems in the primary mathematics curriculum and made presentations by considering the problem solving stages.
- Students' presentations in the classroom were performed according to Polya's (1945) problem solving stages. The presentations performed included the problems in different subjects.
- Attention was paid to the use of a different strategy in each question.
- At the end of the each solution, what kind of other strategies these problems could be solved with and their ways of solution was discussed.
- The order in the Table-1 was followed in the solution of the problems.
- Table-1 was created considering Polya's (1945) problem solving stages. Each problem was evaluated by considering the stages of understanding the problem, choosing the involved strategies, applying the strategies and evaluating it.
- The problems determined and mathematical thinking scale were applied to the students again at the end of the term.

Table 1. Evaluation Criterion Table of Problem Solving Stages



Analyses of the Data

Analyses of the problems developed by Posamentier and Krulik (1998) were analyzed according to Table-1 considering Polya's (1945) problem solving stages. Percentage and frequency values of the answers the students gave to each problem were calculated. In terms of the two problems applied at the beginning of the term and for the same problems applied at the end of the term, analyses were made with the following arrangement.

1. At the stage of understanding the problem, determining the givens and unknowns in the problem.

2. At the stage of choosing the strategy, choosing the strategy such as making a systematic list, guessing and testing, Drawing a diagram, finding a pattern, solving an equation-inequation, estimating, solving a simplier problems, studying retrospectively, Making a table, logical reasoning. The chosen strategy's being right or wrong, and making right or wrong solutions with the chosen strategy.

3. At the stage of applying the strategy, determining whether there is a suitable solution for the chosen strategy and whether the chosen strategy was used correctly.

4. At the stage of assessing the solution, the stages of what we did and where did it?, why we did it?, control of the solution, offering another way for solution were respectively were tried to be revealed.

At the stage of the solution of mathematical thinking scale, the analysis was made with the independent groups t-test.

FINDINGS AND COMMENTARIES

"Does the subject of Problem Solving have an effect on the students' skills of understanding the problem, determining and applying the strategy, and evaluating it ?" The Findings Pertaining to the Problem

The First Problem: The answers the students gave at the beginning and end of the term to the problem of "There are 5 persons in a room. What is the probability of at least two's birth days' being same?" were provided in Table-2.

Understanding of the Problem

At the stage of understanding the problem, the evaluations regarding the given and the unknowns were presented in Table-2.

Table-2. The answers given at the stage of understanding in the first problem at the beginning and end of the term.

Understanding the Problem	Answers	Beginning of Term		End of Term	
		Frequency	Percent	Frequency	Percent
Givens	Right	5	%22	17	%100
	Wrong	18	%78	0	%0
Unknowns	Right	5	%22	16	%70
	Wrong	18	%78	1	%30
Writinig of Questions text	Right	7	%30	0	%0
	Wrong	16	%70	0	%0

According to Table-2 revealed that the students who were taught the problem solving stages for two course hours at the beginning of the term had difficulty in the stage of the understanding of problem. While 22% of the students determined the given correctly, 78% of them determined them wrong. It becomes clear at this stage that the problem cannot be solved without understanding it. It was determined that most of the students (78%) stated the unknown wrong. It was established that what the students understood from understanding the problem was that the problem text had to be written as it was. However, 70% of those writing the problem text wrote the problem sentence in a wrong way. This findings shows how important the place of problem solving stage is in understanding the problem. It is difficult to solve correctly a problem which is not understood well.

All of the students (100%) stated the stage of indicating the given in a correct way. 70% of the students provided positive statements in the proposition of the unknown. When the answers at the end of the term are examined, it becomes clear that there is a positive increase in students' statements concerning the given and unknown.

Selection of the Strategy

At the stage of the selection of strategy, the strategies the students chose, their frequency and percentage values are presented in Table-3.

Table 3.The answers given at the selection strategies stage in the first problem at the beginning and end of the term.

Selection Strategies	Beginning of Term		End of Term	
About Strategies	Frequency	Percent	Frequency	Percent
Estimating	5	%22	3	%18
Making A Systematic List	5	%22	1	%6
Making A Table	4	%17	4	%24
Solving An Equation-İnequation	3	%13	3	%18
Guessing And Testing	3	%13	3	%18
Finding A Pattern	2	%9		
Logical Reasoning	1	%4	3	%18
Drawing A Diagram			1	%6
Solving A Simplier Problems			2	%12

Table-3 is formed of the strategies the students chose in the selection of strategies, which is one of the problem solving stages. It is understood from the percentage and frequency values that the students chose making a systematic list, guessing and testing, drawing a diagram, finding a pattern, solving an equation-inequation, estimating, solving a simplier problems, studying retrospectively, making a table, logical reasoning. It was revealed at the stage of practice whether the strategies the students chose at the beginning and end of the term were suitable for the solution of problem. The evaluation pertaining to the stage of practice is presented in Table-4.

Application of the Strategy

At this stage, it was checked whether the chosen strategy was applied correctly.

Table 4. The answers given at the stage of strategies implementation in the first problem at the beginning and end of the term.

Strategies İmplementation	Beginning of Term		End of Term	
	Frequency	Percent	Frequency	Percent
Right İmplementation	3	%13	8	%47
Wrong İmplementation	20	%87	9	%53

The above table revealed that at the beginning of the term, the students could not apply the strategies they chose. It becomes apparent that the students failed to apply the strategies indicated in Table-3 at the stage of solving the problem (87%). This finding is an indication that the students who cannot understand the problem will not be able to reach the solution with the strategies they randomly choose.

Evaluation of the Solution

The data pertaining to evaluation of the solution, which is the last of the problem solving stages are provided in Table-5.

Table-5. The answers given to the stage of evaluation of the solution in the first problem at the beginning and end of the term.

Evaluation of the solution	Beginning of Term		End of Term	
Right Evaluation	Frequency	Percent	Frequency	Percent
	3	%13	8	%47
Wrong Evaluation	20	%87	9	%53

Table-5 is the indication that the problem cannot be solved without understanding it. At the end of the term, there was a positive increase in the correct evaluation of problems. It can be clearly seen in Table-5 that the problem cannot be solved with the chosen strategy without understanding it.

The Second Problem: The answers which the students gave to the problem of "There are two peg tops below. We have two numbers at hand derived from (a) and (b). What is the probability of the sum of two numbers' being even?" at the beginning and end of the term are provided below.

Understanding of the Problem

At the stage of understanding the problem, the evaluations regarding the given and the unknowns were presented in Table-6.

Table-6. The answers given at the stage of understanding in the second problem at the beginning and end of the term.

Understanding the Problem	Answers	Beginning of Term		End of Term	
		Frequency	Percent	Frequency	Percent
Givens	Right	10	%43	17	%100
	Wrong	13	%57	0	%0
Unknowns	Right	22	%96	17	%100
	Wrong	1	%4	0	%0

The above table demonstrates that at the stage of understanding the problem, the correct answer given (43%) at the beginning of the term increased (100%) at the end of the term. It can be said that the students were efficient in correctly expressing the given for the subject taught during the term. It is reported that after solving various problems, the correct answers were 100% in the expression of the unknown.

Selection of the Strategy

Understanding the problem is the first and fundamental stage for the solution. The person who understands the problem well will reach the solution by choosing the correct strategy. Below are the evaluations aiming at choosing the strategy, which is the second stage.

Table 7. The answers given at the selection strategies stage in the second problem at the beginning and end of the term.

Selection Strategies	Beginning of Term		End of Term	
About Strategies	Frequency	Percent	Frequency	Percent
Making A Systematic List	9	%39	3	%18
Making A Table	13	%57	7	%40
Guessing And Testing	1	%4	3	%18
Solving An Equation-İnequation			2	%12
Drawing A Diagram			2	%12

When we look at the Table-7, we notice that at the beginning of the term, students chose three strategies, but at the end of the term they chose five strategies. What is important at this stage is to determine whether they use the selected strategy or not in the solution of the problem.

Application of the Chosen Strategy

Below is the evaluation of the chosen strategy.

Table 8. The answers given at the stage of strategies implementation in the second problem at the beginning and end of the term.

Strategies İmplementation	Beginning of Term		End of Term	
	Frequency	Percent	Frequency	Percent
Right İmplementation	3	%13	17	%100
Wrong İmplementation	20	%87	0	%0

While number of the students who applied the strategy correctly at the beginning of the term was 3 (13%), at the end of the term it was found that number of those who applied the strategy correctly was 17 (100%). A very marked decline was observed in the number (0%) of students who failed to apply the strategy s/he chose (87%). At this stage, the application of chosen strategy correctly will surely lead to correct solution. Below are the correct solutions and evaluation of the problem.

Evaluation of the Solution

Evaluation of the applied strategy at the beginning and end of the term is presented below.

Table-9. The answers given to the stage of evaluation of the solution in the second problem at the beginning and end of the term.

Evaluation of the solution	Beginning of Term		End of Term	
	Frequency	Percent	Frequency	Percent
Right Evaluation	3	%13	17	%100
Wrong Evaluation	20	%87	0	%0

Table-9 demonstrates that at the end of the term, 17 students (100%) made the evaluations correctly. In the evaluations made at the beginning of the term, 3 students (13%) made the evaluations correctly, and 20 students (87%) made them wrong. The increase at the end of the term reveals that the process of instruction was sustained effectively.

"Does the subject of problem solving have effect on the students' mathematical thinking levels?"

The Findings Pertaining to Problem Status

In the study, the students were subjected to "Mathematical Thinking Scale" before and after the course of "Problem Solving in Mathematics". Whether the data were suitable for normal distribution was determined before the analysis of the data obtained. Normality test of the X_i observations of n unit, which was randomly selected from the population exhibiting normal distribution, is obtained with W test statistics. W test statistics shows difference at the interval of $0<W\leq1$. The values close to 1 show that the variable has a normal distribution, on the other hand, the values close to 0 show that the variable does not have a normal distribution (Özdamar 2004). The Table for the normal distribution of the data is presented below.

Table 10. Normality Distribution Table

Shapiro-Wilks Test Statistics	W Statistics
Pre Test	0,928
Post Test	0,902

As can be understood from the Table-10, the data pertaining to mathematical thinking scale show a normal distribution. Because the data show normal distribution, t-test analysis was conducted in order to determine whether there is a significant difference among the mathematical thinking scores. The analysis performed is presented in Table-11 below.

Table 11. Pre Test-Post Test Mathematical Thinking Scale t-test Table

Test of sort	N	$\overline{X}$	Ss	p
Pre Test	23	91,7826	4,6412	0,000*
Post Test	17	107,4706	5,2454	

The table above brought out that the "Mathematical Thinking Scale", which was applied before and after the course of "Problem Solving in Mathematics", produced meaningful results. This meaningfulness ($p<0,05$) show that post-test results ($\overline{X}$ =107,4706) were higher than the pre-test results ($\overline{X}$ =91,7826). As a result, it becomes clear that the process treated in the course of "Problem Solving in Mathematics" increased the mathematical thinking level of the students.

RESULTS AND DISCUSSION

The results regarding the research in which the effect of the subject of "Problem Solving in Mathematics" on the students' problem solving skills and their mathematical thinking levels was investigated are expressed below according to each problem.

It became evident that the students had difficulty in the phase of understanding the problem at the beginning of the term. At the stage of understanding the problem, 22% of the students specify the given correctly, while 78% of them specify them wrong. The students expressed most of the unknown in a wrong way. In addition, it was determined that the students understood the stage of understanding the problem in a way that they had to write the text of problem as it is. However, 70% of those who wrote the text of problem wrote the sentence of problem wrongly. At this stage, it becomes clear that the problem cannot be solved without understanding it. This result indicates how crucial the place and importance of the problem solving stage is in the understanding of problem. It is difficult to solve correctly a problem

that was not understood well. Thus, it is necessary that the given, unknown and required be specified in a well-understood problem.

The students solved the problems, which were applied at the end of the term, by understanding the stage of understanding the problem. At the stage of stating the given, all of the students (100%) used a correct statement. 70% of the students made correct statements in the expressions of the unknown. When answers at the end of the term are examined, it becomes clear that there were increase in the students' answers in the statement of given and unknown when compared to their answers at the beginning of the term. This result reveals that the understanding of given, unknown and required would lead the students to the solution of problem.

At the stage of selecting the strategy, it was established that the number of strategies the students chose at the end of the term was more, while at the beginning of the term, they chose less strategies. It is understood from the percentage and frequency values that students making a systematic list, guessing and testing, drawing a diagram, finding a pattern, solving an equation-inequation, estimating, solving a simplier problems, studying retrospectively, making a table, logical reasoning in both application.

At the stage of applying the strategy, it became evident that at the beginning of the term, students failed to apply the strategy they chose. 87% of the students could not applied the suggested strategies at the stage of solution of the problem. This result is the indicator of the fact that the students who did not understand the problem cannot reach a solution through the strategies which they chose randomly. At the end of the term, there were positive increases in the accurate assessment of the problems.

In the consequences of the second problem, on the other hand, the accurate answers given at the beginning of the term (43%) were understood to increase at the end of the term (100%) , at the stage of the comprehension of the problem. It can be said that the lessons handled during the term were effective in terms of students in expressing the givens in an accurate way. It is understood from the frequency and percent values that after solving some problems in expressing the unknown, correct answers were 100 %.

Understanding the problem is the first and basic stage of the solution. The student who understands the problem well will reach a solution choosing the involved strategy.

At the beginning of the term, the students were observed to choose the strategies of making a systematic list, making a table, and guessing and testing, while at the end of the term they were observed to choose the strategies of making systematic list, making a table, guessing and testing, solving an equation-inequation, and drawing a diagram. The period in the term can be said to increase the selection of the strategy. What is important at this level is to determine whether the selected strategy can be used in the solution of the problem.

At the beginning of the term, it was found that the number of students who practiced the selected strategy accurately was 3 (13%), while at the end of the terms this number was found to be 17 (100%). A very clear decrease is understood in the number of students who cannot practice (87%) the selected strategy (0%). At this stage, the accurate performance of the selected strategy undoubtedly leads to the correct solution.

At the end of the term, it was revealed that 17 students (100%) accurately performed the step of problem assessment. In the evaluations made at the beginning of the term, 3 students (13%) made it correct but 20 students (87%) made it wrong. The increase in the proportions at the end of the terms disclose that the lesson course was conducted effectively.

The general conclusion of the study is that the education provided for the purpose of increasing problem solving skill has become effective in the development of this skill, and in the increase of their level of the selection and use of proper strategy. Pressley (1995) put forward that the students having received education for problem solving strategies were more successful and had higher problem solving skills than those not having informed about these strategies. The consequences of certain studies carried out similarly also showed that after having been taught several problem solving strategies, students could learn and use these strategies, their mathematical success increased and problem solving skills developed (Altun 2005; Arslan 2002; Faubion 2001; Ghunaym 1985; Lee 1982; Sulak 2005; Yaşar 2010; Yazgan 2002; Yazgan and Bintaş 2005; Yıldızlar 1999). From this point of view, the consequences of the performed studies are parallel to those of our study.

In the consequence of "Scale of Mathematical Thinking" , the results of posttest ($\overline{X}$ =107,4706) were determined to be higher than pretest results ($\overline{X}$ =91,7826) . The solutions of the problems made positive contributions to the students' skills of thinking. Students start thinking first in the process of understanding at the level of solving each problem. The students who understand the problem start to think mathematically at the level of moving on to the solution by selecting the correct strategy. As Tall stated (1995), the individual, in mathematical thinking, knows that when he looks , as a whole, at the content which he formed in the mind, it is easier for him to remember it. In the process of problem solution, the students knows that he can go the solution when he gets the solution steps of the problem.

Schoenfeld (1992) states that for the mathematical thinking to develop, the process should be evaluated well and the practices are necessary. Mathematically-thinking student, in the process of problem solving, should complete the process in the most proper way by using the problem solving steps.

Suzuki (1998) states that students generally tend to memorize the operations and the formulas while learning mathematics. Yet, a student knowing the problem solving steps will understand the problem and start the solution selecting the proper strategy. While using the mathematical knowledge, students will both start to think mathematically and solve the problems by understanding them. At this level they will get rid of the tendency of memorizing.

When the results on mathematical thinking are considered, it has been concluded that the period for the lesson of problem solving produced positive effects on the students' skills of mathematical thinking. It can be said that the applied process affected the mathematical thinking skill positively. This arrived result revealed that as the result of the problems practised in the class of problem solving and the steps of problem solving, mathematical thinking can be developed.

As a consequence, the class "Problem Solving in Mathematics" was revealed to have positive effects on students' problem solving skills and mathematical thinking levels. Also, it was determined that after students had learnt problem solving strategies, they realized that that they could use many strategies in problem solving. Therefore, it was concluded that the class "Problem Solving in Mathematics" had the students gain problem solving strategies and increased the students' level of mathematical thinking.

REFERENCES

Altun, M. (1995). *A Study on the Problem Solving Behaviors of 3,4 ve 5. Grade Primary Students.* Unpublished Phd Dissertation, Hacettepe University, Ankara.

Altun, M. (2005). *Mathematics Teaching for Education Faculties and Primary Teachers.* Bursa: Aktüel Publications.

Arslan, Ç. (2002). *A Study on the Levels of Using Problem Solving Strategies of Elementary Seventh and Eigth Grade Students.* Unpublished Master Dissertation. Uludağ University, Social Science Institution, Bursa.

Baki, A. (2006). *Mathematics Education From Theory to Implementation.* Trabzon: Derya Publishing.

Baykul,Y., (1999). *Mathematic Teaching in Pimary 1-5 Grades.* Anı Publishing: Ankara.

Bransford, J.D. & Stein, B.S. (1993). The Ideal Problem Solver (2nd Ed). New York: Freeman

Burkhardt, H. (1994). Mathematical applications in school curriculum. In T. Husén & T. N. Postlethwaite (Eds.), The international encyclopedia of education (2nd ed.) (pp. 3621-3624). Oxford/New York: Pergamon Press.

Charles, R., Lester, F., & O'Daffer, P. (1994). *How to Evaluate Progress in Problem Solving* (5th ed.). Virginia: The National Council of Teacher of Mathematics.

Dewey, J. (1991). *How we think*. New York: Prometheus Books, Buffalo.

Faubion, K. A. (2001). *If you already knew how to do it, it would be a problem: middle school problem solving strategies.* MS. Thesis, The Faculty of Pacific Lutheran University.

Ersoy, E. (2013). *Qualitative Research Methods.* S. Baştürk (Ed.), *Scientific Research Methods* (339-374). Ankara: Vize Publishing.

Ersoy, E. (2012). *The effects of High Level Thinking Abilities on Emotional Acquirements in Learnig Process Based on Problem.* Phd Dissertation. Dokuz Eylül University. Social Science Institution. İzmir.

Ghunaym, G. (1985). *An investigation of the effect of instruction in the structure of problem solving strategies on students' performance. PHD.* Thesis The Florida State University.

Güçlü, N. (2003). "The Problem Solving Skills of High School Managers*", National Education Journal*, 160, 272-300.

Hatfield, M.M., Edwars, N.T., Bitter, G.G. (1997). *Mathematics methods for elementary and middle school teachers* (3th Ed.). Boston: Allyn&Bacon.

İsrael, E. (2003). The Relationships of Problem Solving Strategies, Achievement Level, Socioeconimic level and Gender. Master Dissertation, Dokuz Eylül University, Education Science Institution, İzmir.

Lee, S.,K. (1982). Fourth graders' heuristic problem-solving behavior. *Journal for Research in Mathematics Education*, 13 (2), s.110.

National Council of Teachers of Mathematics (2000). *Principles and standards for school mathematics.* Reston, Va. NCTM.

Miller, C.M. (2000). Student-Researched Problem-Solving Strategies. *Mathematics Teacher*, 93 (2), 136-138.

Mintzberg, H. (1994). The rise and fall of strategic planning. Basic Books.
Özdamar, K. (2004). Statistical Data Analysis through packaged Softwares-1. MINITAB-NCSS-SPSS. Broaden 5. Press. Eskişehir: Kaan Publishing.
Polya, G. (1945). How to Solve It, Princeton NJ: Princeton U. Press.

Posamentier, A.S., Krulik, S. (1998). "Problem-Solving Strategies For Efficient And Elegant Solutions: A Resource For The Mathematics Teacher" Thousand Oaks; Corwin Press.

Sağlamer, E. (1980). *Mathematics Teaching in Primary.* M.O.N.E: İstanbul.

Şahin, M. (2007). The importance of efficiency in active learning. *Turkish Science Education Journal,* 4(2), 61-74.

Schoenfeld, A.H. (1992). Learning to Think Mathematically: Problem Solving, Metacognition and Sense-Making in Mathematics. In D. Grouws (Ed.), Handbook for Research on mathematics Teaching and Learning (pp. 334-370). New York: MacMillan.

Sulak, S. (2005). The Effects of Problem Solving Strategies in Primary Mathematics Lessons on the Achievement of Problem Solving. Master Dissertation. Selçuk University, Social Science Institution, Konya.

Suzuki, K. (1998). Measuring "To Think Mathematically": Cognitive Characterization of Achievement Levels in Performance-Based Assessment. University of Illinois at Urbana-Chanpaign The Graduate College. Doctor of Philosophy.

Polya, G. 1945. *How to solve it: A new aspect of mathematical method*, Princeton, USA, Princeton University Press.
Pressley, M. (1995). *Cognition, Teaching ,Assesment.* New York: Harper Collins College Publishers.

Salaria, N. (2012). Meaning of the Term Descriptive Survey Research Method. *International Journal of Transformations in Business Management,* 1(6).
Tall, D. (1995). Cognitive Growth in Elementary and Advanced Mathematical Thinking. Plenary Lecture, Conference of International Group fort the Psychology Mathematics, Recife, Brazil, July, (Vol I, pp. 161-175).
Tanrıseven, I. (2000). *The use of Dramatization Method as Problem Solving Strategy in Mathematics Teaching.* Unpublished Master Dissertation. Marmara University, Enstitute of Education Science, İstanbul.
Willoughby, M. (1991). *Mathematics Education for a Changing World.* Virginia: Association for Supervision and Curriculum Development.
Yaşa, E. (2010). *The Effects of Teaching of Problem Solving Strategies supported worksheets on the Achievement of the Students.* Unpublished Master Dissertation. Eskişehir Osmangazi University, Enstitute of Science, Eskişehir.
Yavuz, G. (2006). *The Effects of Teaching Problem Solving Strategies in 9th Grade Mathematics Lessons on the Emotional Properties* Unpublished Phd Dissertation. Dokuz Eylül University,Enstitute of Education Science, İzmir.
Yıldırım, C. (2004). *Mathematical Thinking*. İstanbul: Remzi Kitabevi. Yazgan, Y. (2002). An Investigation on the Levels of Use Problem Solving Strategies of Fourth and Fifth Grade Primary Students. Master Dissertation. Uludağ University, Social Science Institution, Bursa.
Yazgan, Y. ve Bintaş, J. (2005). the Levels of Use Problem Solving Strategies of Fourth and Fifth Grade Primary Students: A Teaching Experiment. *Hacettepe University Education Faculty Journal,* 28: 210-218.
Yıldızlar, M. (1999). The Effects of Teaching Problem Solving Behaviors to Primary *1., 2. and 3. Grade Students on Achievement and the Attitude of Mathematics.* Phd Dissertation. Hacettepe University, Social Science Institution, Ankara.