PROBLEM SOLVING AND ITS TEACHING IN MATHEMATICS

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Abstract: Purpose of this study is to determine problem solving skills of primary mathematics preservice teachers in mathematics teaching. This research was carried out with the 3rd year students studying in the department of elementary mathematics teaching at Samsun Ondokuz Mayıs University. Research method was determined as case study, one of the qualitative methods. In the study, the students were taught for 13 weeks (39 hours) Polya's (1945) problem solving stages that are composed of 4 stages and the problem solving stages were introduced in order to improve their problem solving skills. In the research, two problems developed by Posamentier and Krulik (1998) and semi-structured interview form developed by the researcher were used as data collection tools. In the analysis of the data, solutions of the problems applied were examined considering Polya's (1945) problem solving steps. The findings obtained via the solutions of applied problems and via the semi-structured interview form were established with the percentage and frequency values. As a result, it became evident that subject of problem solving has a positive effect on the development of mathematics teachers' problem solving skills.
Keywords: Mathematics Teaching, Problem Solving, Problem Solving Stages.

1. Introduction

Important mathematics concepts and procedures can be best taught through problem solving (Van De Walle, 2007; 37). Focusing on problem solving in lessons develops the students' high level thinking. For this reason, students perform self-learning in mathematisc lessons with problem solving process. Problem solving plays an important role in mathematics education and most of learning is an occour as a result of problem solving process.

Problem solving is an integral part of all mathematics learning, and so it should not be isolated from mathematics program (NCTM, 2000). Schoenfeld (1992) declared that there were numerous unaddressed issues dealing with problem-solving instruction and assessment. The need for seek to answer questions concerns what actually takes place in problem-centered classrooms (Lester, 1994). Students are encourageds after problem solving process because problem solving contributes to the use of different solutions and development of strategies that students use.

Polya (1945) describes the process of problem solving at four stages, including understanding the problem, determining the strategy, implementing the selected strategy and assessment. At the stage of understanding the problem, the student is expected to state what he understood from the problem and to determine what are the given and unknown in the problem and also to suggest clearly the condition of the problem. At the stage of determining the strategy, the student is expected to determine which steps such as calculation, drawing, etc. to follow in order to reach the requested. The teacher, in this process, can promote the use of different problem solving strategies by writing the all strategies on the board and can enable the student to choose the suitable strategy (Miller, 2000). The following stage includes the application of selected strategy by the student. At the stage of application the selected strategy, the solution should be checked step by step. At the stage of assessment, on the other hand, the student should control whether the solution he made is right and meaningful. During the process of control, it must be fully put forth what has been done and where it has been done.

2. Methodology

Case study method was used in the research. A case study design is employed to gain an in-depth understanding of the situation and meaning for those involved (Merriam, 1998). A qualitative case study is an intensive, holistic description and analysis of a single instance, phenomenon, or social unit (Merriam, 1988, p.21; Merriam, 1998, p.27).

The reason why case study method was preferred in this research is the investigation of problem solving skills and the views pertaining to problem solving process of 26 preservice teachers, studying in the 3rd year of elementary mathematics teaching.

Problem Status

1. Does the subject of problem solving have an effect on students' skills of understanding the problem, determining the strategy, applying and assessing?
2. How is the evaluation of problem solving subject according to students' opinions?

Study Group

The study group was formed of randomly selected 26 students who were taking the subject of "Problem Solving in Mathematics", which is a 3rd year subject of Elementary Mathematics Teaching in the Faculty of Education at Samsun Ondokuz Mayıs University. The selected 26 students solved the problems. Of these students, an interview was made with nine of the students who were randomly selected.

Data Collection Tools

In the study, two problems arranged by Posamentier and Krulik (1998) and translated into Turkish, and "Student Interview Form for Problem Solving", developed by the researcher were used. The data collection tools were explained below in detail.

The Problems

The problems asked to the students are included below.

Problem 1. Evelyn, Henry and Al play a certain game. The player who loses each round must give each of the other players as much money as the player has at that time. In Round 1, Evelyn loses and gives Henry and Al as much money as they each have. In Round 2, Henry loses, and gives Evelyn and Al as much money as they each then have. Al loses in Round 3 and gives Evelyn and Henry as much money as they each have. They decide to quite at this point and discover that they each have $24. How much money did they each start with?

Problem 2. A jeweler makes silver earrings from silver blanks. Each blanks makes 1 earring. The shavings left over from 6 blanks are then melted down and recast to form another blank. The jeweler orders 36 blanks to fill an order. How many earrings can be made from the 36 blanks?

Student Interview Form for Teaching the Problem Solving Strategies

In the research, "Student Interview Form for the Problem Solving Strategies", developed by the researcher, was used as data collection tool. In the subject of problem solving, four main categories were formed while writing the interview items aiming at establishing the students' problem solving processes. The major categories arranged are as follows: (1) problem solving stage, (2) thinking skill, (3) practice and (4) assessment of the process. 11 questions were written on the interview form intended for problem solving strategies. The data obtained in the interview were recorded with a recorder. Permission was taken from those whom the interviews would be made with prior to the interviews. Having established the main categories, sub-categories were tried to be fixed. The sub-categories were determined by the two reseachers in terms of the reliability of the study. The inventory of interviews conducted was made to determine the sub-categories. The sub-categories were formed in accordance with the common results which both researchers obtained in terms of each question of item.

Another reliability step aimed at interview techniques, on the other hand, is the consistency in the process of writing out the talks recorded during the interview (Kvale, 1996: 236, quoted by Türnüklü, 2000). It is necessary to check the consistency of analysis in this process for the mistakes that may arise in the analysis of the tapes obtained after the interview to be reduced. Therefore, it is required that the consistency in the process of both analyses should be checked by analysing a part of the conversation, recorded to the tape, at two different times. To that end, agreement percentage formula can be used.

$$P = \frac{N_a \times 100}{N_a + N_d}$$

P: Agreement Percentage; N_a = Quantity of Agreement, N_d = Quantity of Disagreement

Data Collection

The data were collected in the subject of “Problem Solving at Mathematics” in the first term of 2013-2014 academic year. The period of practice of the subject was completed in 13 weeks. In the course of the lesson, Polya’s (1945) problem solving stages were told to the students. A number of problems were solved towards the end of the term and it was discussed what kind of strategies the problems could be solved. During the term, students formed groups in threes, and they solved some problems in the elementary mathematics program by considering the problem solving stages, and then they made presentation. The students’ presentations in the class were carried out according to Polya’s (1945) problem solving stages and in each question, they took care of employing different strategy. The presentations made cover the problems at different topics. At the end of each solution, it was discussed what kind of strategies the problems could be solved with and their ways of solutions. Each problem was evaluated considering the stages of understanding the problem, selecting the involved strategy, and applying and assessing the strategy. The specified problems and interview forms were applied to the students at the end of the term.

3. Results

The findings and comments as regards the problem condition of “Does the subject of problem solving have an effect on students’ understanding the problem, determining the strategy, applying and assessing the solution” are presented below.

First Problem

Understanding of the Problem

At the stage of understanding the problem, the evaluations regarding the given and the unknowns were presented in Table-1.

Table 1. The frequency and percent values at the stage of understanding in the problem

Understanding the Problem	Answers	Frequency	Percent
Givens	Right	23	%88
	Wrong	3	%12
Unknowns	Right	21	%81
	Wrong	5	%19

Table-1 reveals that students solved the problem by comprehending the understanding stage of the problem. At the stage of expressing the data correctly, 88 % of the students did it correct, while 81 % of the students express the unknown correct. This discovery revealed that students were able to identify the given easily than the unknown.

Selection of the Strategy

At the stage of selecting the strategy, the frequency and percentage values the students chose are presented in Table-2.

Table 2. The frequency and percent values at the selection strategies stage

Selection Strategies	Frequency	Percent
Working Backward	12	%46
Solving An Equation-İnequation	8	%31
Making A Table	5	%19
Making A Systematic List	1	%4

Table-2 suggests that the students selected the strategies of working backward (12 solutions), solving an equation-inequation (8 solutions), making a table (5 solutions) and making a systematic list (1 solution).

Application of the Strategy

At this stage, it was checked whether the selected strategy was applied correct or not. Whether the strategies the students selected were suitable or not for the solution of the problem was revealed at the stage of application. The assessment pertaining to application stage is provided in Table-3.

Table 3. The frequency and percent values at the stage of strategies implementation

Strategies Implementation	Frequency	Percent
Right Implementation	25	%96
Wrong Implementation	1	%4

The above table revealed that 96 % of the students applied the strategy they selected in acorrect way. This findings demonstrates that the correct solution of the problem will be achieved via suitable strategy after understanding it.

Evaluation of the Solution

The data pertaining to the evaluation of the solution, which is the last of the stages of problem solving are given in Table-4.

Table 4. The frequency and percent values the stage of evaluation of the solution

Evaluation of the solution	Frequency	Percent
Right Evaluation	25	%96
Wrong Evaluation	1	%4

Table-4 revealed that rate of the students who reached a correct solution via the strategy he chose and who could evaluate the solution (f=25) was quite high (96%). As a result, it can be said from the correct evaluation (96%) that students are able to answer what we did and where?, what we obtained?. It was concluded that the students who reached the correct solution with the strategies they selected were successful at the stage of evaluation.

Second Problem

Understanding of the Problem

The data pertaining to the evaluation of the solution, which is the last of the stages of problem solving are given in Table-5.

Table 5. The frequency and percent values at the stage of understanding in the problem

Understanding the Problem	Answers	Frequency	Percent
Givens	Right	26	%100
	Wrong	0	%0
Unknowns	Right	25	%96
	Wrong	1	%4

The above table reveals that all of the students understood the problem. All of the students (100%) determined the given in a correct way, while 96% of them expressed the unknown correctly.

Selection of the Strategy

At the stage of selecting the strategy, the frequency and percentage values the students chose are presented in Table 6.

Table 6. The frequency and percent values at the selection strategies stage

Selection Strategies	Frequency	Percent
Making A Systematic List	8	%31
Drawing A Diagram	6	%23
Making A Table	6	%23
Finding A Pattern	3	%12
Guessing And Testing	2	%7
Logical Reasoning	1	%4

Table-6 suggests that the students chose many strategies in the selection of the strategy. It becomes evident that students selected the strategies of making a systematic list, drawing a diagram, making a table, finding a pattern, guessing and testing and logical reasoning. This finding revealed that students tendency to select different strategy increased.

Application of the Strategy

At this stage, it was checked whether the selected strategy was applied correct or not. The assessment pertaining to application stage is provided in Table 7.

Table 7. The frequency and percent values at the stage of strategies implementation

Strategies Implementation	Frequency	Percent
Right Implementation	24	%92
Wrong Implementation	2	%8

Table-7 reflects that the number of students who applied the selected strategy correct (f=24) was extremely high and then those who applied it (f=2). This finding reveals that thanks to the correct strategy selected in the process of solving the problem, correct applications were made.

Evaluation of the Solution

The data pertaining to the evaluation of the solution, which is the last of the stages of problem solving are given in Table 8.

Table 8. The frequency and percent values the stage of evaluation of the solution

Evaluation of the solution	Frequency	Percent
Right Evaluation	22	%85
Wrong Evaluation	4	%15

The above table indicates that 85% of the students made correct evaluations. This finding reflects that the selected strategy after understanding the problem produced correct results. It becomes evident that the students who reached a correct solution with the selected strategy and who were able to evaluate the solution (85%) completed the problem solving stages successfully.

"How is evaluation of the subject of problem solving according to the students' opinions?" The Findings pertaining to Problem Status

Interviews were carried out to determine to what extent the students learned the problem solving stages taught in the subject of problem solving in mathematics. In the research, semi-structured interviews were conducted with 9 students at the end of the term. The entire interviews carried out with the students were recored to tape and all of the interviews were transformed into texts. In consequence of the interviews conducted with the students, the answers obtained aiming at major categories and sub-categories were encoded and their frequency and percentage values were calculated.

Prior to encoding the data to the previously-arranged categories, encoding reliabilities of the people to make the encoding procedure were calculated as 91%. The consistency of analysis was also checked in this process for the mistakes that might arise in the analysis of tapes obtained after the interview to be reduced. To that end, one part of the conversation was analyzed at two different times and the consistency in each of the analyses was calculated. Agreement percentages according to the major categories formed in terms of reliability of the research was determined as 91% at the stage of problem solving stage, 96% at the category of thinking skill, 88% at the category of application and 89% at the category of evaluating the process. The findings pertaining to the interviews carried out in the subject of problem solving in mathematics are presented in Table 9.

Table 9. Findings Related to Problem Solving Course

Main Categories	Sub Categories	Codes	Frequency	Percent
Problem Solving Stage	Understanding of the Problem	Givens Unknowns Text Problems	6 5 5	%67 %55 %55
	Selection Strategies	Different selection strategies Selection appropriate strategies Ease of solution Quick solution	5 3 6 9	%100 %67 %55 %33
	Strategies Implementation	Relationships between concepts Different solution way Prior Knowledge Modelling	4 7 7 3	%78 %78 %44 %33
	Evaluation of the Solution	Permanent learning What we learn Come to fruition Mental processes	4 4 6 7	%78 %67 %44 %44
Thinking Skill	Mathematical Thinking	Mathematics language Process skills Reasoning	7 9 8	%100 %89 %78
Application	Assessment	Reach a solution understanding Problem solving with stages Get rid of memorization Adaptation to daily life Point of view	4 5 3 4 5	%55 %55 %44 %44 %33
Assessment of process	Solving Process	Creating Formulas Examine in detail a problem Mathematical inference Put away prejudice Decrease in calculation error Ease of learning Review textbooks Appropriate selection strategies	4 9 7 6 5 4 8 6	%100 %89 %78 %67 %67 %55 %44 %44

The Findings and Commentaries Regarding the First Main Category

The sub-categories pertaining to the main category of “Problem Solving Stages” were analyzed in the sub-categories of “Understanding of the Problem”, “Selecting the Strategy”, “Strategies Implementation” and “Evaluation of the Solution”.

When students’ views regarding the sub-category of “Understanding of the Problem”, 67% of the students expressed their opinions as regards the fact that the given should clearly be specified at the stage of understanding of the problem. 55% of the students stated that they were easily able to move on to the selection of the involved strategy after understanding of the unknown. The students who remarked that problem text should be very clear (f=5) reported that they reached the solution by understanding the problems.

When students’ views regarding the sub-category of “Selecting the Strategy”, 100% of them express they move on to the different strategies, 67% of them they can make analysis with the suitable strategy, 55% of the students states that they can reach the solution easily with the strategies they chose and 33% of them say they make rapid solutions.

When students' views regarding the sub-category of "Strategies Implementation", 78% of them states they can link up the inter-conceptual relationships, 78% of them says they can find different ways of solution, 44% of the students remarks that they use preliminary information and 33% of them indicates that they can make modelling while applying the strategy.

When students' views regarding the sub-category of "Evaluation of the Solution", 78% of them remarks they attained permanent learning, 67% of them says they analyze what they learned during the solution, 44% of the students reports that they can reach the solution easily while evaluation the solution and 44% of them states they can make mental processess.

When we look at the findings pertaining to the first main category, the students expressed their opinions that problem-solving stage can only be carried out with understanding of the problem, selection and application of the involved strategy and evaluation of the solution.

The Findings and Commentaries Belonging to the Second Main Category

The sub-category belonging to the main category of "Thinking Skill" was examined in the sub-category of "Mathematical Thinking". When students' views for the sub-category of "Mathematical Thinking" were considered, it was observed that 100% of them used mathematical language, 89% of them used process skill and 78% of the students employed skill of logical reasoning.

The Findings and Commentaries Belonging to the Third Main Category

The sub-category pertaining to the main category of "Application" was analyzed in the sub-category of "Evaluation".

55% of them expressed that they solved the problems by understanding, 55% of them said they found the solution thanks to he problem solving stages, 44% of the students reported that they were able to associate the evaluations with the daily life while making them, 44% of them said they got rid of memorizing and 33% of them stated that their perspectives to problems became more positive,

The Findings and Commentaries Belonging to the Fourth Main Category

The sub-category pertaining to the main category of "Evaluation of the Process" was analyzed in the sub-category of "Process of Solution". 100% of them to be able to create formula, 89% of them to evaluate the problems in a more detailed way, 78% of them to be able to make mathematical inferences, 67% of them to make less mistakes in the processes, 67% of them to get rid of bias at the stage of evaluation of the solution, 55% of them to pass the learning process easily while making evaluation, 44% of the students was reported to finish the process correctly if they select the suitable strategy and 44% of them not to examine their course books in detail this much.

4. Discussion

According to the results obtained, the students determined the given and unknown in the second problem in a correct way to a large extent, while they exhibited less success in writing the given in the first problem. It can be said that this result may change according to the degree of students' distinguishing the given and unknown depending on the difficulty of the problem. In addition, it was concluded that the students were able to detect the given more easily than the required in determining the given and unknown data in the problem. This situation may be due to students' being less in the habit of questioning what is exactly unknown in the problem.

Because students are prone to reaching the result immediately by employing the numbres in the problem and to moving on to the following problem while solving the problems (Erbaş and Okur, 2010), they start to make process without focusing on what is the unknown.

A number of research results carried out similarly also suggested that after the students had been taught various problem solving strategies, they learned and used these strategies, their success in mathematics increased and their skills of problem solving improved (Adibnia and Putt, 1998; Altun, 2005; Arslan, 2002; Artzt, and Armour-Thomas, 1998; DeBellis and Goldin, 2006; Faubion, 2001; Fan and Zhu, 2007; Ghunaym, 1985; Lee, 1982; Sulak, 2005; Wilson, Fernandez and Hadaway, 1993; Yaşar, 2010; Yazgan, 2002; Yazgan and Bintaş, 2005; Yıldızlar, 1999). In this sense, the studies carried out show parallelism with the research results.

5. Conclusions

It is observed that when looking to the strategies the students selected while solving the given problems, it is seen that the strategies show difference. The strategies of making a table and making a systematic list were used in both of the problems, while in the first problem they employed the strategies of studying retrospectively and writing equality. It is observed that the taught strategies enriched the ways of solutions. On the other hand, when considering the degrees of the preference of these strategies, in the first problem, the students at most applied the strategies of working backward study (12 solutions), solving an equation-inequation (8 solutions), making a table (5 solutions), respectively, whereas they employed the strategy of making a systematic list at the least (1 solution). In the second problem, the students at most selected the strategies of making a systematic list (8 solutions), drawing a diagram (6 solutions), making a table (6 solutions), finding a pattern (3 solutions), guessing and testing (2 solutions), respectively, whereas they applied logical reasoning at the least (1 solution). In this case, it can be said that the strategies the students selected change depending on the structure of the problem. In addition, the students' levels of adopting and apprehending the strategies may affect their the frequency of preferring the strategies. As a result, student is prone to select the strategies he understands best. Besides these, it was observed that some students tried to solve the problems without selecting the strategy. However, the number of solutions made applying strategy is more than that of the solutions made without applying any strategies in both problems. Students' trying to solve the problems without using any strategy is due to their encountering the indicated strategies for the first time and yet failing to internalize them.

When students' ways of applying the strategy they select are examined, it was concluded that in both problems, more than half of the solutions were derived through suitable strategies and each of the problems the selected strategies were applied correctly by 92% - 96%. This finding, on the other hand, reveals that problem solving skill is learnable (Larkin, 1980; Chi, Feltovich & Glaser, 1981; Azai & Yokoyama, 1984; Verschaffel, De Corte & Lasure, 1999; Altun & Sezgin Memnu, 2008) and if suitable learning environment is provided, that's to say, if students are continually and systematically subjected to problem solving process as in this research, this acquired skill brings success in the solution of another problems.

In literature, there are plenty of researches supporting that problem solving has significant contribution to person's cognitive development (Verschaffel et al., 1999; Dochy, Segers &Bossche, 2003; Gijbels, Dochy & Bossche, 2005; Özsoy, 2005; Akinsola & Awofala, 2008; Çalışkan, Sezgin Selçuk & Erol, 2010; Tüysüz, Tatar & Kuşdemir, 2010). In addition, there also some solutions made wrongly even though correct strategy was selected. This situation demonstrates that selecting correct strategy does not that it will lead to correct answer. Rudder (2006), Erbaş and Okur (2010), in the studies they carried out, obtained the results supporting our findings. The results derived for the two problems at the stage of evaluation of the problem indicate that the students are successful in evaluating the answers they provided.

Recommendations

According to the results of interviews, it was concluded that the subject of problem-solving affected students' skills of problem-solving stages, their thinking processes, and their applying and evaluating the problem in a positive way. The actively participation of the students, who received the education of problem-solving, in the lesson was determined with their positive views to the subject.

Presley (1995) set forth that the students receiving education for problem solving strategies are more successful as compared with those not being informed about these strategies and their problem solving skills are high.

The process of solving problems is now completely interwoven with the learning; children are learning mathematics by doing mathematics (Van De Walle, 2007; p.39). Problem solving stages contributes to the development of students. Students use various and different solution strategies. This situation encourages students.

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