THE HORIZON OF CONNECTIONS BETWEEN MATHEMATICS AND ART: OBSERVATIONS FROM A TEACHER EDUCATION COURSE

Özlem Çeziktürk, Assist. Prof. Dr.
Marmara University
Ozlem.cezikturk@marmara.edu.tr

ABSTRACT
Since the 2004 spring, mathematics and art course has been formed for pre service math and science teachers of primary and secondary level. This paper includes a summary of the feedback from the implementation of this course and of what we have learned from connections and their effects on mathematics learning. Students showed increased appreciation for mathematical art and increased mathematical thinking skills. Every student left the course with a portfolio. They could interrogate their artefacts with their classmates. They built and saw 3D models of platonic solids via modular origami. Their thinking and reasoning with abstract mathematics extended beyond what can be seen as in 4D. They reasoned about geometrical relationships as an expert mathematician. Patterns flourished as when they were least expecting. Seeing became multi-dimensional. Connections led to some emerging themes from the course gains as imagery, visualization, representations, cognition and connectionist models, patterns, mathematical thinking.
Keywords: mathematics and art, origami, fractals, patterns, dimension

1.0. INTRODUCTION

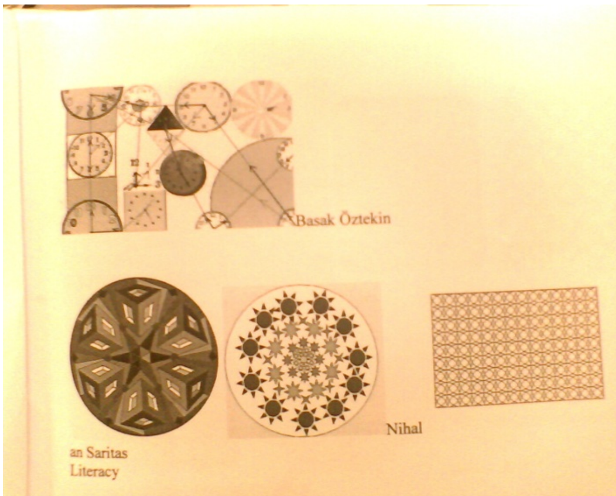
There is a workshop series that has been running for the last decade and lasted by two extraordinary academicians: Dr. Annalisa Crannel and Dr. Marc Frantz. I encountered with their Viewpoints 2003 workshop (Crannell, Frantz 2011) invitation while I was lost via very interesting connections over the Internet. While this paper is not only on what they do, since the whole idea is based on what I have learned from that workshop, it would be very fair to start with their ideas. One of them being a very sensitive mathematician and the other being a diverse mathematician with an artistic background, they were very successful in developing a huge enthusiasm with an intellectual spark within us that I will appreciate for the rest of my life. They had the passion for what they were doing and they let us have at least a part of it as well.

In that workshop, we have learned on one point/ two point perspective and how we should view art with that knowledge at hand, fractals; how we could generate them, what they are based on (iterations, self-similarity, and very distinct dimensions), modular origami (from a workshop attendant who had a passion to disseminate the idea of origami to all parts of the society), photography, animations and the use of perspective among them, and last but not least was the appreciation of art that we have mastered tremendously. At the beginning, their course notes were helpful in generating new course material.

1.1. The course

The name of the course was a direct application of the two most important things in this course: mathematics and art. Although the name of the course remained constant throughout two universities that we have given this course, and throughout the very large number of students, the contents changed from time to time due to the shift in my interest. Throughout the years more universities provided courses on these topics in Turkey; fractals course on Karadeniz Technical University, Origami course on Başkent University, Paper folding course at Gazi University and the course with the same title “Mathematics and art” in Dokuz Eylül University(Ugurel, Tuncer, Toprak, 2013). Although this paper will focus on our course, it is a must that we should say that it became like a trend. The course name was “SCED 486: Special Topics in science and mathematics teaching course started to be known as “Mathematics and art”. The course had some students who had diverse backgrounds like art and pure mathematics but mainly due to the time and place constraints, and giving this course to only 25 students at most each time (besides one time to 90 students in a separate university), some preservice elementary teaching students and also some students from arts and sciences faculty were able to take this course.

Fig 1. Example of student artefacts



The course was on *connections.* Connections that shape mathematics, connections that shape art were the main course outline. However, students were the main locomotives of the course. They have created, they have analyzed, and they have interrogated (figures 1,2,7,8,and 9). They have even helped us to develop the course. Each semester, I have used what we have gained from the students in the previous semester regarding the new connections, the connections overheard, or the connections just ignored. In the last semester, the contents of the course even included some excursions into the mathematics to deal with some anxiety and fear problems related with mathematics itself.

Students' enthusiasm reached to the staff of the department and they even became a part of a presentation of modular origami and kirigami (art of cutting paper) demonstration to the whole university on the Teachers' day. Connections were created; connections were developed via students' minds.

For the assessment part, some criteria were established as creativity, harmony, the degree of mathematics embedded /used, aesthetics, modularity, difficulty, etc. In semester where we have taught this course in Marmara University, we have included also another part that we have not before. We have made the students to assess each other's work with the criteria that we had. We have made use of the differentiated instruction so that students were able to bring a developed model of what they were interested in. Some of the students chose to go one point ahead of an excursion; some students created a copy of an artistic expression and some students chose to develop an original piece of their own.

Fig 2. Example of student artefacts

After the first year, instead of a stable course format, we have designed the course with respect to what the students brought to the class. That is students who were good at computers helped us to see how we could use computers to generate images that could challenge everyone to think what messages they could be carrying out. Musician students helped us to see different ways to be a musician with interesting musical instruments, with notes and without notes, and only being interested in patterns. Hence, we chose to use a dynamic course material with flexible format in each different semester. We have used art to learn math and we have used math to learn art.

1.2. Art and Mathematics: Connections across…

Only some pure mathematicians like Poincare believed that it includes aesthetics as its soul. Mathematics is known to be a cold course which lacks sympathy, which lack emotion. Hence, it becomes less understandable and less achievable. But the reason is esthetic, sympathetic, and pedagogical side is less considered while teaching (Dursun, 2012). Dursun (2012) also mentions about the necessity to give minimal exactness (nothing unnecessary is included) and maximal benefit (since it is simple but it can be used in everywhere). In the course, unnecessary connection is decided by its following connection. For example, golden ratio is not left by its definition or by the sunflowers but its sequential importance of powers is searched and new connections are achieved.

The course is not for their eyes only, but also for those who are the true searchers of the beauty of mathematics. The name of the course lacks to give its possible richness of the topics that could be mentioned. In the literature, most of the connections between mathematics and art were already discussed in full but there are some new issues or some new areas that deserves full attention. These new connection areas may provide new horizons not only for the teachers but also for their students. Fractals, origami, mathematical poetry, animations, Islamic patterns, crotchet, quilts are some of them. There are also other topics such as perspective, collage, painting, mathematical 3-D modeling, advertisements, graphical design, illusions, etc.

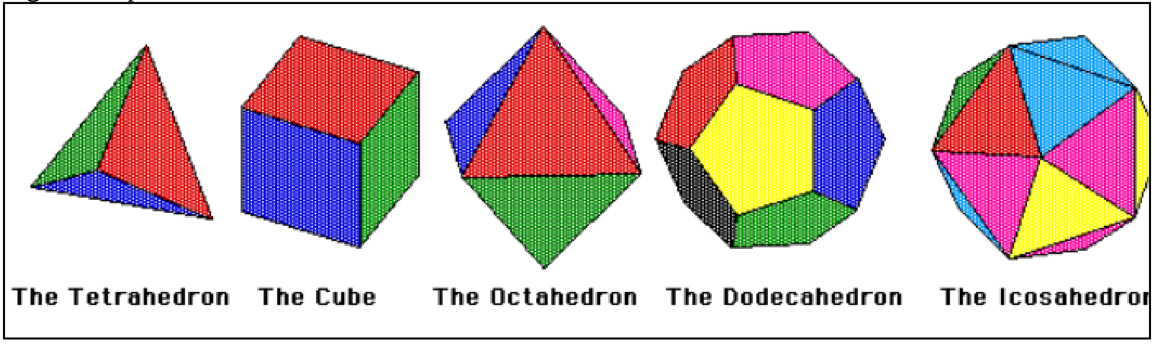
The literature on mathematics teacher education states that when teachers are given a chance to think mathematics as art, they tend to teach mathematics with esthetics and with caution. Some teachers may already have beliefs related to math as art then all they need is a starting point (Baydar, & Bulut, 2002). National Council of Teachers of Mathematics of USA (NCTM, 2000) mention mathematics education should employ mathematics as art. Similarly, our curriculum for both elementary and secondary math education aims students making connections between math and art and development of their sense of aesthetics. Not only mathematics curriculum but also geometry curriculum asks for connections to be built between square, pentagon, hexagon and art. In 10^{th} grade and in the previous grades all geometric connections are flourish from Turkish Islamic patterns(Ugurel, Tuncer, Toprak, 2013). In their study, they have studied 43 preservice teacher from secondary education. Out of these 27 of them were able to make connections in their prepared lesson plans on mathematics and art connections as good or medium. They have used case study method with some descriptive statistics and it looks like a one and only study on this subject regarding some numbers. From the students' written lesson plans, fractals, mostly used referring to sequences and infinity concepts, golden section is used in sequences, sets, functions, and specifically on the squares. Escher's art on impossibles is used for paradoxes, 2D/3D or for limit and infinity (Ugurel, Tuncer, Toprak, 2013). As a result, preservice teachers' connections are left in a restricted area. Ugurel, Tuncer and Toprak proposed (2013) inter disciplinary approach for mathematics education and lesson plan development with teachers are necessities.

Most of the aforementioned constructs are known for centuries but the connections are forgot to be pointed for a very long time and people started to think that these are all but nothing to do with mathematics. As a matter of fact, even people who are good at these art constructs believe that they do not employ mathematics or they could not be able to do mathematics on their own. In this manner, it is conjectured that if we can educate at least some teachers on these connections this could help both to the lowering of the anxiety level towards mathematics and also help new generations to use more *mathematical themes* as well as some mathematical considerations for the advancement of these art constructs. The mathematical themes that emerged over time follows: imagery, visualization, representations, cognition, patterns, and mathematical thinking.

1.2.1.Imagery; Platonic solids (figure 3) are mainly five polyhedra that Platon iniated: tetrahedra, cube, octahedron, dodecahedron and icosahedron. We have started using sonobe modules to generate a cube (six modules), then we demonstrated stellated dodecahedron (in which each face was having a pyramid on it) and stellated icosahedron with many sonobe module variations. Students were able to compose different polyhedra with various modules. First, they worked in groups, and then they worked independently. Imagery came into the situation where some of them went beyond intended and asked questions like "*what happens when….*" or "*what is the next step if we build something like this…*" .

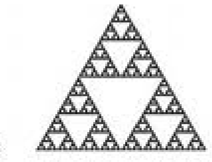
Reasoning with the numbers of corners, sides, faces and dissection of polyhedra gets easier with the modular origami pieces at hand. Students drew impossible figures and analyzed dimension construct. Perspective drawing on given lattices was experienced with these figures. Some of the students used this chance to color and visualize the 3D in 2D. These tasks became ill-defined tasks for them to reason mathematically.

Fig 3. Five platonic solids



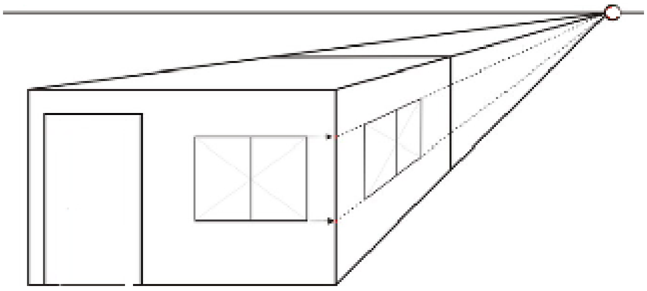
Fractals are mathematical shapes of the chaos theory. A fractal is a self-similar figure with iterations and a special dimension. They have taught us that dimensions can be decimal numbers, nature has a written mathematics underneath, fractal generation is a creative yet calculated manner etc. Students imagined connections that happen without intentions. For example, if we color even numbers in Pascal's triangle, we end up with Sierpinski triangle (figure 4) pattern. It has a Hausdorff dimension of (log3/log 2). Another fractal creates a tree shape when initiated from Phytagorian proof. These types of connections are imagery sharpeners, and they help students see what they have not noticed before. Karakuş (2010) proposed the same thing with preservice teachers on 3D fractal cards which enable dimension concept development.

Fig 4. Sierpinski triangle



1.2.2. Visualization; What we see is not what we get from mathematical connections. They are everywhere. Students need to be trained to be able to see what is hidden and not easily visible. For this reason, mathematics pathways should be a topic for lifelong learning. Here, perspective topic helped us. Technical drawing made us to see it from front, back and sides. Perspective with one vanishing point made us see it from a corridor, perspective with two vanishing points made us see it within a building corner and outside of a building corner (figure 5). We saw buildings, paints, tiles, boxes, from different perspectives. Putting in a perspective is just a transformation with points in three dimensions. Hence, we faced with the transformational geometry as a connection.

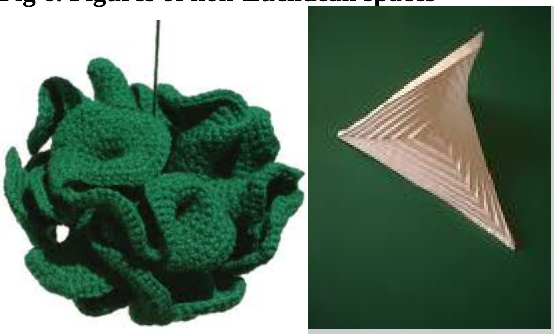
Fig 5. One point perspective



1.2.3.Representations; We have used origami and special techniques of paper folding to represent things that we cannot show otherwise. Four dimensional space and beyond is a question for many preservice mathematics teachers. Non Euclidean spaces (spaces or subspaces where Euclid laws do not work) are not easy to be seen as well. With one corner of them attached, we have inserted one more triangle to six triangles within a hexagon. Similarly, we have glued hexagons to a heptagon sides and saw how the surface was made of hills and depths. With crocheting (if we increase one knot by double in each row), we can represent hyperbolic surfaces of non-Euclidean space by visuals. In the figure below, paper hyperbolic surface is created by folding a square paper from its diagonals and folding from its grids one outside one inside on each produced triangles inside (fig 4).

Similarly, we can cut stripes from concentric circles with the same thickness and we can glue them one outside one inside next to each other to experience another non Euclidean space. Baki (2001) argues that Euclid geometry is not helpful to teach theoretical and physical world by itself at the same time. Non Euclidean geometries on the other hand give the opportunity of shapes of mountain lines, or the branches of a fern or the curls of clouds and snowflakes.

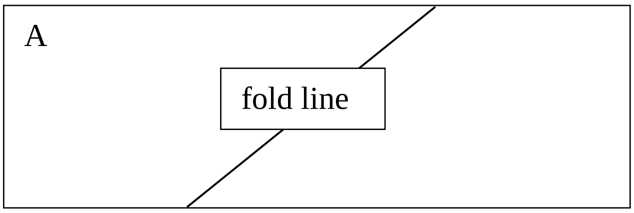
Fig 6. Figures of non-Euclidean spaces



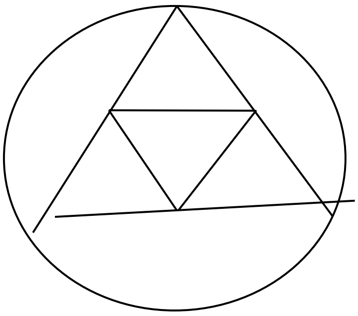
crotcheting and paper folding creates non-Euclidean spaces

1.2.4.Cognition *and the connectionist models***;** In most of the activities, we have chosen the ones that intrigue mathematical reasoning most. *Origamics* (Haga et al 2008) is a branch of origami which deals with some interesting folds and the mathematical connections achieved. Some questions are asked in well known journals like Bilim ve Teknik in Turkey.

Question: If we fold two opposite corners of a rectangle paper (A and B), what is the length of the resultant fold line?



Question: how we can transform a circle into a half cone?



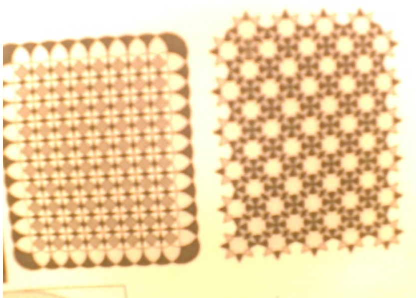
These questions produced an array of answers, and they produced more than one parallel answers. Interesting fact was students' different reasoning styles and the way we were able to see it from their array of answers. Research on these answers will follow this paper immediately. Cognitive theories of learning stress the algorithm and concept learning(figure 6). With these two questions and with many more examples, students identify different possible algorithms and interrogate concepts like rectangle, diagonals, sides, angles, similar angles, circle, angles in a circle, prisms, height of a prism, etc.
After a while, students create their own questions as well. These types of questions are also the requirements of the recent curricula since they ask for reasoning with patterns, identifying connections between different concepts and problem formation.

Fig 7. Student made crotchet helix

1.2.5.Patterns; Nature is a great source of patterns. Fibonacci numbers are on sun flower's seeds. But they also produce a series that associates with the golden number (1,618). With the powers of the golden number, we have encountered with a mathematical series that connects to Fibonacci numbers. We asked for what may come next? Or where it connects to? And even, some inductive proof came into the consideration.

Fig 8. Student made helix

Fig 9. Student made patterns



Patterns on the doors, windows, and buildings of Islamic architecture is at first amazing, and they flourish connection with the geometry of triangles, squares, rectangles, plane division problems and even with plane symmetry groups (Schattschneider 1978). 17 wallpaper groups of Alhambra Palace help us to deal with dimension concept via impossible solid figures of Wiltshire (1997). Hand crafts carry an array of special examples of connections. For example, they can help us to analyze the spirals, helixes.

1.2.6.Mathematical thinking*;* Mathematics should be taught emphasizing mathematical thinking and reasoning as a process. This view has been mostly supported by recent mathematics education standards and mathematics education researchers. Even new curricula stress the importance of making students think like a mathematician (Turkish Ministry of Education 2005). Process rather than product and content are stressed (Mamona-Downs, & Downs 2002). Burton pointed to mathematicians thinking in terms of the big picture and with using the connectives and reflecting on structure (undated, cited in Mamona-Downs, & Downs, 2002).

2.1.HYPOTHESIS AND METHODOLOGY

In this longitudinal study, it was hypothesized that a course like this could create awareness towards connections between art and mathematics and with other branches. In addition, it was thought that understanding that there is an infinite array of connections could foster more need to master mathematics and more need to be exposed to art in general.

Although, it is hard to see the results of the course material on students since they are disseminated worldwide after their graduation from two universities that this course was given. This course material grew by itself over the time. More connections are revisited, more connections are recognized. But above all, I as the teacher of this course and as the primary student of this course became the most interesting product of this course. In this paper, I will also state some side products that came with me as in the form of increased understanding of learning by connections and mathematical reasoning formation before, during and after the artifacts that are created by what we learn from this course throughout the years pass by.

It is a longitudinal study since it has been over the course of many years. It is also a case study since as each student teacher and their unique products could be named as different cases, me as both the teacher and the student of this course could be taught as a case for learning by connections revisited.

2.1.1.The course outline

Not all but some art uses mathematics as its structure. *Fractal art* is the collection of all images that are generated by hand or by computer and which corresponds to the some specific complex and iterated functions. *Perspective* can be distorted and this may give us some ways to analyze TV news, advertisements, etc. As much as there is false perspective which is generated by distorted one, there is also true but unexpected perspective as in the case of relative look into an object and with different types of object (sphere, cylinder, etc.) There is one point, two point and there point perspective and studying them may help understanding the connections between architecture, mathematics and art. Foundations and basic folds of *origami* is followed by *modular origami* (figure 10) to see the part whole relations, to investigate the five platonic solids and their variations (Erktin 2005). As the years pass by, we reached to a point where we needed to understand more of what we produce. And *origamics* became an intensive part of the course, so that a new course can be formed from its artifacts and reasoning manners. *Musical notes* are in progression of equal *tone* intervals. In each division, there is the same measurement of notes, and they are a good resource of reasoning with decimals, and rational numbers. Creation of *poems* about mathematics is fun, on mathematics is challenging, and of mathematics is a light of *structuralism* inside.

Fig 10. Modular origami (Sonobe modules and miscellaneous modules)

From the poem of Nazım Hikmet(1901-1963) ;
“This is the last day of your journey, last!
In these four days, the wagon
How many times it was filled, how many times it was emptied?
How many passengers were left behind?
We have smiled and looked after the ones who left and the ones who left their places to the new comers…” we have reasoned about the functions, word problems, and even created little poetry.

Blaise Pascal (1623-1662) thought us limit…”There are three points where the knowledge comes face to face; one is the premature, pure and natural illiteracy, the other is the border line to which only big thinkers can reach when they walk from the pathways of the scientific knowledge, and it is where they realize that they know nothing.”

3.1.RESULTS
As an artifact of the course, the following constructs are built by juxtaposition. Art literacy level of both me and the students increased immensely. Students realized that creation is a process that one needs to elaborate on and think. This made them to prepare themselves for working like a mathematician. They brought pieces of their mathematical art work almost every week (as a course grading procedure, they were graded “+” for their small art works, but after a while their bonuses exceeded almost the number of weeks in a semester. They studied mathematics more than they were accustomed to. They became to appreciate more artistic and mathematically enriched pieces of art by the weeks passing. Students were more creative during the course without a question. Some of the worksheets required collaborative and cooperative studying. Their group skills jumped to a higher level. Identification and reasoning with patterns is a crucial issue in everyday life. The patterns they saw were mathematical art patterns of ISAMA and BRIDGES (two mathematical art conferences which stress originality and high level mathematics behind). Their confrontation with these made them critical of their own creations. Students found themselves to teach what they learn to their siblings, brothers, sisters, students, friends, families. And we know that teaching is half way of learning. They were cognitively guided by their level of interest and cognitive immersion into the mathematical content and this also acted for them as a therapy.

3.2.CONCLUSION
Both art and mathematics are thought to be for some elite groups like for the most intelligent or for the wealthiest. I think this course has showed to me and to students that connections revisited may help us create some new ways for underrepresented students to go forward both in mathematics and in art.

It is important to mention that this course was not alone. Throughout the years, there have been at least two other universities in Turkey who took this type of courses into their academic programs. It is believed that in the following years, more research and observations from these other courses could be seen in the journals and conferences so that the number and the variety of these types of math and art connection courses flourish.

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