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NUMERICAL SIMULATING FOR RAIN-WIND INDUCED VIBRATION OF INCLINED CABLES

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Abstract: In this paper, the mechanism of rain-wind-induced vibration of inclined cables is further studied based on the theory of multi-degree-of-freedom cable element (Ma, 2003). After both influence of axial and out-of-plan vibrations of cable are neglected, the in-plan vibrations of cable are studied and the influence of water rivulet on aerodynamic forces are considered. A two degree-of-freedom nonlinear model of the coupling system is developed and the governing equations for the vibration amplitude are derived. Then Hurwitz discriminant is used to evaluate the kinematic stability of the system. When the damper or the stiffness of the system is negative, self-excited vibrations of the cable will occur, which is the essence of rain-wind-induced vibration. Because there exits a limited cycle, the free vibration of the nonlinear system has a steady amplitude. By means of the harmonic balance method, the dynamic responses of the system are calculated. Numerical example is given to show that the developed model is reasonable and effective.

Keywords: Rain-wind induced vibration, inclined cable

INTRODUCTION

Rain-wind-induced vibration of an inclined cable is a severe vibration with large amplitude, which might cause fatigue damage in short periods and should be mitigated in engineering. Such dynamic behavior is a solid-liquid-wind interaction problem with complicated mechanism. The forming and the moving of water rivulet on the surface of a cable in wind and rain circumstance changes cable section and aerodynamic forces; the latter, on the other hand, affects the vibration of the cable and the water rivulet. The interactions among the cable, the water rivulet and aerodynamic forces induce self-excited vibration of the system, namely rain-wind-induced vibration. Because of so many infecting factors and the nonlinear characteristics, the mechanism of such vibration is complex and difficult to analyze.

Hikami and Shiaishi (1988) firstly observed rain-wind-induced vibrations on Meikonishi Bridge in Nagoya, Japan, where the amplitudes of inclined cables were observed up to 55cm under wind of velocity 14m/s. During the vibration, a water rivulet was observed to appear on the lower surface of the cable, oscillating in circumferential direction with the same frequency of the cable. Further wind tunnel experimental research showed that the cable oscillations were mostly of single mode in the vertical plane and that the formation position of water rivulets depended on mean wind velocity. Based on further wind tunnel test results and field measurement results, Matsumoto et al. (1992, 1995, 2003) concluded that the formation of upper water rivulet and the axial flowing might be the inducement of rain-wind-induced vibrations. Bosdoginni and Oliver (1996) compared the tunnel test results between fixed water rivulet model and moving water rivulet model and indicated that the position, not the moving, of upper water rivulet was the primary cause of the vibration.

Compared with experimental research, analytical study is relatively limited. Yamagushi (1990) proposed a two-degree-of-freedom galloping model, considering the cable as a horizontal rigid cylinder. After Peng (2001), Xu, Wang (2003) and Wilde, Witkowski (2003) described the movement of rivulet as simple harmonic circumferential oscillating at the frequency of the cable, the plane model was further studied, and the analytical results were compared with those from wind tunnel tests. It turned out that such analytical models could capture main dynamic features of inclined cylinders with either moving rivulet or artificial fixed rivulet. However, because the plane model assumes that the vibration of cable and rivulet is same along cable length, it might not be applicable to an integral cable. A multi-degree-of-freedom cable model was developed by Ma in 2003, where the inherent modes were used to simulate the dynamic curve of the cable, and the oscillation of rivulet was explored. However, as a further study, this paper presents a two degree-of-freedom nonlinear model of the coupling system after both influence of axial and out-of-plan vibrations of the cable being neglected. Then Hurwitz discriminant is applied to evaluate the kinematic stability of the model and the harmonic balance method is employed to calculate the amplitude of the cable.

Theoretical model

The mechanical model is shown in Fig.1, where x is chord direction of the cable, and θ is the inclination angle. S is the average tension along the chord. z_1 is the local orientation in the moving direction of the cable. $\beta(x,t) = \beta_0(x) + \gamma(x,t)$ is the instant position of the rivulet, $\beta_0(x)$ and $\gamma(x,t)$ are the initial position and oscillating angle of the rivulet, respectively. $U(x)$ and $U_{RE}(x,t)$ are wind velocity and relative velocity.

As the cable oscillations are mostly of a single mode in the vertical plane, the dynamic curve of the cable can be simulated through its inherent vibration in-plane mode functions. After both influences of axial and out-of-plan vibrations are neglected, the multi-degree-freedom model of Ma (2003) may be simplified as

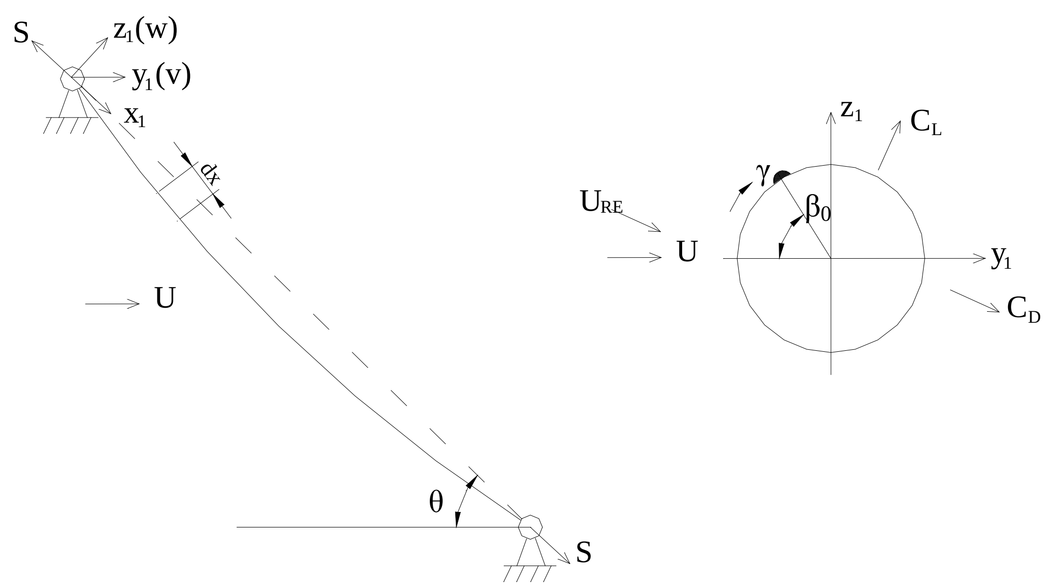

Figure. 1 Model of the system of cable and rivulet

$$m_{11}\ddot{q}_w + c_{11}\dot{q}_w + k_{11}q_w + k_{12}q_w^2 = f_w \quad (1)$$

$$m_{21}\ddot{q}_w + m_{22}\ddot{q}_\gamma = g_1\ddot{q}_w q_\gamma \quad (2)$$

where $m_{11} = M\int_0^l \phi_w^2 dx$, $m_{21} = mD/2\int_0^l \phi_w \phi_\gamma \cos\beta_0 dx$, $m_{22} = m\left(\frac{D}{2}\right)^2 \int_0^l \phi_\gamma^2 dx$,

$g_1 = mD/2\int_0^l \phi_\gamma^2 \phi_w \sin\beta_0 dx$, $f_w = \int_0^l F_{z1}\phi_w dx$, $k_{11} = -S\int_0^l \phi_w'' \phi_w dx + \frac{EA}{l}\int_0^l z_1''\phi_w dx \int_0^l z_1'\phi_w' dx$,

$\phi_\gamma(x) = \cos\beta_0(x)\phi_w(x)$, $\phi_\gamma(x)$ is the vibration mode of the rivulet, ϕ_w is the in-plane mode function of the cable, c_{11}, *l* and *D* are the damper coefficient, chord length and the diameter of the cable, k is the rotation stiffness of the rivulet, $z_1(x)$ is the initial curve function of the cable, $w(x,t)$ is the vertical dynamic displacement, *EA* is the section stiffness, *M* and *m* are the mass per unit length of the cable and the rivulet. F_{z1} is the wind pressure component in $z_1$direction:

$$F_{z1} = \rho D U_{RE}^2 [-C_D \cdot \sin\alpha^* + C_L \cdot \cos\alpha^*]/2 \tag{3}$$

where $C_L(\alpha), C_D(\alpha)$ are lift and drag coefficient, respectively and may be expressed as

$$C_D(\alpha) = C_{D0}(x) + C_{D1}(x)\cdot(\gamma - \alpha^*) + C_{D2}(x)\cdot(\gamma - \alpha^*)^2 + C_{D3}(x)\cdot(\gamma - \alpha^*)^3 \tag{4}$$

$$C_L(\alpha) = C_{L0}(x) + C_{L1}(x)\cdot(\gamma - \alpha^*) + C_{L2}(x)\cdot(\gamma - \alpha^*)^2 + C_{L3}(x)\cdot(\gamma - \alpha^*)^3 \tag{5}$$

$$\alpha^* = \arctan(\dot{w}/U) \tag{6}$$

where $\gamma(x,t) = q_\gamma(t)\phi_\gamma(x)$, $w(x,t) = q_w(t)\phi_w(x)$.

Combining equation (1), (3), (4), (5), we get

$$\ddot{q}_w + 2\xi_w\omega_w\dot{q}_w + \omega_w^2 q_w + k_{12}/m_{11}q_w^2 = \rho D U^2/m_{11}\cdot g_w \tag{7}$$

where

$g_w(q_\gamma, \dot{q}_w) = a_0 + a_1\dot{q}_w + a_2\dot{q}_w^2 + a_3\dot{q}_w^3 + a_4 q_\gamma + a_5 q_\gamma^2 + a_6 q_\gamma^3 + a_7\dot{q}_w q_\gamma + a_8\dot{q}_w^2 q_\gamma + a_9\dot{q}_w q_\gamma^2$

$a_0 = \int_0^l C_{L0}\phi_w dx$, $a_1 = \int_0^l (-C_{D0} - C_{L1})\phi_w dx/U$, $a_2 = \int_0^l (C_{D1} + C_{L0}/2 + C_{L2})\phi_w dx/U^2$,

$a_3 = \int_0^l (C_{D0}/2 - C_{D2} - C_{L1}/2 - C_{L3})\phi_w dx/U^3$, $a_4 = \int_0^l C_{L1}\phi_w dx$, $a_5 = \int_0^l C_{L2}\phi_w dx$, $a_6 = \int_0^l C_{L3}\phi_w dx$,

$a_7 = \int_0^l (-C_{D1} - 2C_{L2})\phi_w dx/U$, $a_8 = \int_0^l (2C_{D2} + C_{L1}/2 + 3C_{L3})\phi_w dx/U^2$, $a_9 = \int_0^l (-C_{D2} - 3C_{L3})\phi_w dx/U$,

ω_w, ξ_w are the circular frequency and damper ratio of the cable.

Similarly, equation (2) may be rewritten as

$$m_{21}/m_{22}\ddot{q}_w + \ddot{q}_\gamma = g_1/m_{22}\ddot{q}_w q_\gamma \tag{8}$$

Judgment for the stability of the governing equation

The linearized equation of the system is

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = 0 \tag{9}$$

where $\mathbf{M}=\begin{bmatrix}1 & \\ m^* & 1\end{bmatrix}$, $\mathbf{C}=\begin{bmatrix}c_{11} & 0\\ 0 & 0\end{bmatrix}$, $\mathbf{K}=\begin{bmatrix}\omega_w^2 & k_\gamma\\ 0 & 0\end{bmatrix}$, $\mathbf{q}=\begin{Bmatrix}q_w\\ q_\gamma\end{Bmatrix}$, $k_\gamma=-\rho DU^2/m_{11}a_4$, $c_{11}=2\xi_w\omega_w-\rho DU^2/m_{11}a_1$, $m^*=m_{21}/m_{12}$.

The characteristic polynomial of equation (9) is

$$G(\lambda)=\det(\mathbf{M}^*\cdot\lambda^2+\mathbf{C}^*\cdot\lambda+\mathbf{K}^*)=\lambda^2(\lambda^2+e_1\lambda+e_2) \tag{10}$$

where $e_1=c_{11}$, $e_2=\omega_w^2-m_\gamma^* k_\gamma$

Equation (9) remains stable, when all the eigenvalues of equation (10) has negative real part. According to Hurwitz criterion, the condition requires that the inequalities

$$g_i>0 \qquad i=1,2 \tag{11}$$

be satisfied, where $g_1=e_1$, $g_2=e_1e_2$. Since e_2 is positive for usual wind velocity, the stability of equation (11) only depends on the sign of e_1, or c_{11}. When damper of the system $c_{11}$is positive, the system remains stable; otherwise rain-wind-induced vibration occurs. The result is similar to that of the plane models and wind-tunnel test results.

Calculations for cable amplitude

As the nonlinear terms of response appear in the right side of equation (7), the amplitude of cables will not increase unlimitedly. There exists steady vibration with constant amplitude for the system when rain-wind-induced vibration occurs. The harmonic balance method is employed in the paper.

The dynamic response of steady vibration of the cable can be expressed as

$$q_w=A_w\cos\omega t \tag{12a}$$

$$\dot{q}_w=-\omega A_w\sin\omega t \tag{12b}$$

$$\ddot{q}_w=-\omega^2A_w\cos\omega t \tag{12b}$$

Considering phase difference，the dynamic response of rivulet can be expressed as

$$q_\gamma=A_{\gamma1}\cos\omega t+A_{\gamma2}\sin\omega t \tag{13a}$$

$$\dot{q}_\gamma=-\omega A_{\gamma1}\sin\omega t+\omega A_{\gamma2}\cos\omega t \tag{13b}$$

$$q_\gamma=-\omega^2\left(A_{\gamma1}\cos\omega t+A_{\gamma2}\sin\omega t\right) \tag{13c}$$

Substituting (12) and (13) into (7), and considering that coefficients of $\cos\omega t,\sin\omega t$ equal to those on the other side of the equation, one gets

$$A_w(-\omega^2+\omega_w^2)=b_7A_{\gamma1}+\frac{1}{4}b_5\omega^2A_w^2A_{\gamma1}-\frac{1}{2}b_4\omega A_wA_{\gamma1}A_{\gamma2}+\frac{3}{4}(b_2A_w^3+b_9A_{\gamma1}^3+b_9A_{\gamma1}A_{\gamma2}^2) \tag{14a}$$

$$-\omega c_{11}A_w=b_7A_{\gamma2}-\frac{3}{4}(b_3\omega^3A_w^3-b_5\omega^2A_w^2A_{\gamma2})-\frac{1}{4}b_4\omega A_w(A_{\gamma1}^2+3A_{\gamma2}^2)+\frac{3}{4}b_9A_{\gamma2}(A_{\gamma1}^2+A_{\gamma2}^2) \tag{14b}$$

$$-\omega A_{\gamma1}+c_\gamma^*\sqrt{A_{\gamma1}^2+A_{\gamma2}^2}A_{\gamma2}=m^*\omega A_w(1+\frac{3}{8}d_1A_{\gamma1}^2+\frac{1}{8}d_1A_{\gamma2}^2) \tag{14c}$$

$$-\omega A_{\gamma2}+c_\gamma^*\sqrt{A_{\gamma1}^2+A_{\gamma2}^2}A_{\gamma1}=\frac{1}{4}d_1m^*\omega A_wA_{\gamma1}A_{\gamma2} \tag{14d}$$

$$b_i = \rho D U^2 / m_{11} \cdot a_i \quad i = 1, \cdots 9 \tag{14e}$$

$$d_1 = \int_0^l \sin \beta_0 \phi_\gamma^3 dx / \int_0^l \phi_\gamma^2 dx \tag{14f}$$

Thus the nonlinear governing equation (7) is changed into an algebra equation group (14), containing unknowns $\omega, A_w, A_{\gamma 1}, A_{\gamma 2}$.

Example

The longest cable of Yangzi River Bridge is 334m long. The parameters of the cable are as following (Peng 2001): diameter *D*=0.145m, Section stiffness *EA*=1,900,000 kN, mass density M =85kg/m, damper ratio ξ_w =0.1%, inclination angle θ=29^o. The mass density of the rivulet is 0.17kg/m. The stability judgment parameters are shown in Fig. 2 indicating that the rain-wind vibration occurs with the wind speed between 8.4m/s and16.4m/s. The stable amplitudes with different wind speeds are shown in Fig. 3 with comparison of Peng's results (2001).

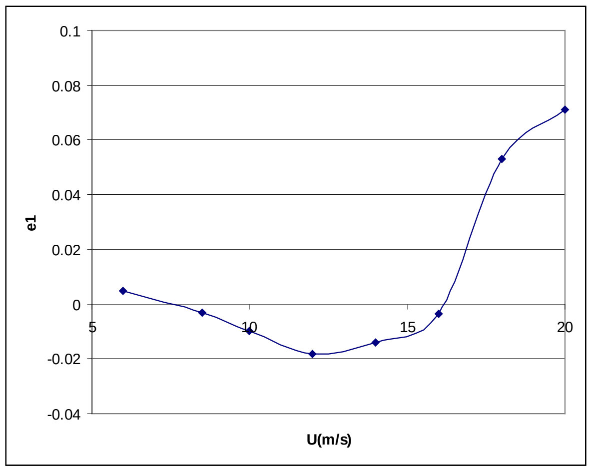

Figure 2. The stability parameters of the system with different wind speed

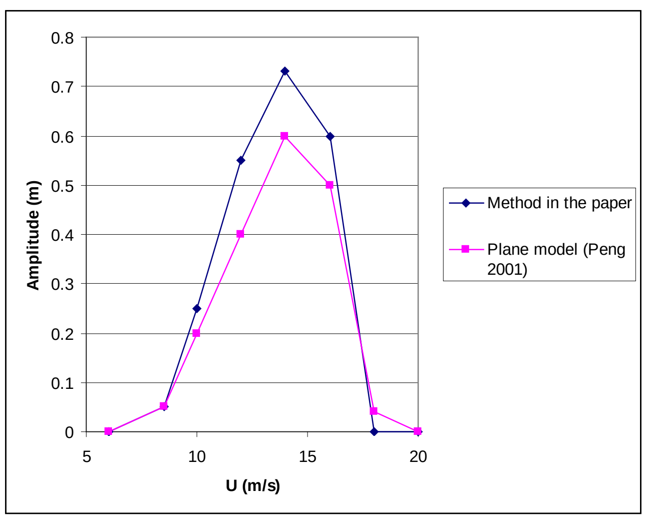

Figure 3. Amplitudes of the cable with different wind speed

CONCLUSION

Considering the in-plane vibration of the cable and the influence of the rivulet position to the aerodynamic forces, a two-degree-freedom model is derived to analyse the rain-wind-vibration of inclined cables and the harmonic balance method is employed to calculate the vibration amplitudes. From analysis above, it may be concluded that the mechanical essence of the complex dynamics phenomenon is a self-excited nonlinear vibration with constant amplitude induced by negative damper.

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