Transverse Thermal Dispersion in Porous Media Under Oscillating Flow

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Abstract: In this study, transverse dispersion thermal conductivity in a porous medium of mono-sized steel balls under oscillating flow has been investigated. Although mode of heat transfer is expected to be convection caused by fluid flow only, conduction and especially dispersion in porous media is very important depending on the solid material, structure and flow medium. The scope of this study is to obtain transverse thermal dispersion conductivity correlations to be used for oscillating flow. For this purpose, a total of 27 sets of heat transfer experiments, each representing a different frequency, flow displacement length and heat input, have been conducted for a porous medium of steel balls of 3 mm in diameters. Considering that the total thermal conductivity is a summation of dispersion conductivity and the medium's thermal conductivity, effective thermal conductivity is found to be a linear function of Peclet number. Thus, it has been possible to estimate effective thermal conductivity of the medium under oscillating flow by measuring heat flux, fluid velocity and calculating radial temperature gradients.

Keywords: Porous Media, Oscillating Flow, heat transfer, dispersion

Introduction

The porous media have been used widely in many engineering fields such as cryocoolers, solid matrix heat exchangers, cooling of electronic equipment and regenerators in order to enhance heat transfer. On the other hand, heat transfer in oscillating flow is a fundamental investigation field. Oscillation-induced heat transport processes maintain an effective heat enhancement comparable with heat pipes. It has many important applications in the compact heat exchangers, cooling processes of nuclear plants, design of Stirling heat machines and heat transport in internal combustion machines. There have been numerous theoretical, numerical and experimental studies on convection heat transfer in porous media and oscillatory flow, individually. Both fluid flow through porous medium and oscillatory flow inside a channel have an effect to enhance heat transfer, so that the investigators have been interested in oscillating flow inside porous medium in recent years.

Heat transfer formulation of porous media with continuum modeling based on a representative elementary volume was improved, and wide information can be found in the studies of Vafai (2005) (Authors referred to: Hsu, Nield and Kuznetsov, de Lemos), Kaviany (1995) and Nield and Bejan (2006).

Zhao and Cheng (1998) presented an extensive review of oscillatory duct flows including heat transfer characteristics. They introduced the similarity parameters of oscillating flow. Correlation equations for the space-cycle averaged Nusselt number in terms of A_o and Rew have been obtained for oscillatory heat transfer in laminar and turbulent reciprocating internal flows.

Leong and Jin (2005) have investigated experimentally the heat transfer of the oscillating flow through a channel filled with aluminum foam subjected to a constant wall heat flux. They introduced a correlation equation for length averaged Nusselt number in term of A_o and Re_ω which was similar to that obtained by Zhao and Cheng for empty channels. They also pointed out that the length averaged Nusselt number for oscillating air flow in porous channel could be up to several times larger than that in empty channel.

Pamuk and Özdemir (2012) conducted heat transfer experiments using steel balls as porous media subjected to oscillating flow where flow medium is water as opposed to the previous studies that used gases. They obtained time and spaced averaged Nusselt numbers as the correlations of A_o Re_ω and Da valid to be a wide range of porous media. Experimental setup used in this study was the same as they used in the current study.

Byun et al. (2006) have investigated the transient behavior of porous media under oscillating flow condition. They presented an analytical characterization of the transient heat transfer in porous media under the oscillating flow condition in their work. They identified two important dimensionless parameters as the ratio of the thermal capacities between the solid and fluid phases and the ratio of the interstitial heat conductance between the phases to the fluid thermal capacity, based on a two-equation model. They obtained the analytic solutions for both the fluid and solid temperature variations, and they classified the heat transfer characteristics between phases into four regimes. They also suggested a criterion for the validity of the local thermal equilibrium in a simple form as the ratio of the two time scales. According this criterion set forth by the authors, the local thermal equilibrium can be achieved when the characteristic time of the porous media is much shorter than the time scale concerning the variation of the boundary condition ($t_p / t_o << 1$).

Dispersion is a complex phenomenon that the tortuosity within the pores cause. Tortuosity is the tendency that causes the fluid to move violently (circulations etc.) in a chatic manner due to the geometry of the porous medium. On the other hand thermal conductivities of both solid particles that constitute porous medium and the fluid decide the amount of heat transferred. An average, or weighted thermal conductivity to present both solid and fluid together which is the medium, is calculated using the porosity which is the volume fraction of the voids (or fluid displacing the voids) and the thermal conductivities of two separate phases. The total conduction heat transfer can be calculated by using the effective thermal conductivity that includes also the dispersion effect which is not easy to calculate since dispersion conductivity can be obtained empirically, or using the various correlations found in the literature. However, these correlations are all for steady flow and mostly for specific conditions. In this paper, we have intended to set forth a correlation for effective thermal conductivity for a porous medium of steel balls under oscillating flow. Following studies have been reviewed to cover literature consisting of stagnant thermal conductivity and dispersion thermal conductivity, thus effective thermal conductivity.

Kuznetsov (2000) investigated the effect of thermal dispersion on fully developed forced convection in a fluid-saturated porous-medium channel bounded by parallel plates. The author found that the effect of accounting for thermal dispersion strongly depends on the Darcy number. If $Da = 10^{-4}$, the effect of thermal dispersion on the Nusselt number becomes visible only for $Re_p > 10^{2.5}$. However, if $Da = 10^{-2}$, thermal dispersion has significant impact on the heat transfer already for $Re_p > 10^{0.5}$.

Hsieh and Lu (2000) performed a parametric study in their work regarding thermal conductivity within the porous media. Their results show that when the ratio of the thermal conductivity of the fluid to the solid increases, the Nusselt number distributions calculated from a one-equation model are getting closer to that from a two-equation model.

Özgümüş et al. (2011) have pointed out that the determination of transverse and axial thermal dispersion conductivities can be classified into three groups as a) Heat Addition/Removal at Lateral Boundaries, b) Uniform Temperature at Inlet Boundary, and c) Heat Addition Inside Bed. Generally, the procedure is almost the same for all these experimental approaches. A temperature gradient is generated in the bed. The temperature is measured at various locations in the packed bed. The equivalent thermal conductivity of the bed can be calculated by using thermal conductivity of fluid and solid and porosity. Thus, if one requires thermal dispersion conductivity, it can be found by subtracting of effective and equivalent thermal conductivities from each other.

Metzger et al. (2004) have found that excellent temperature residuals up to high Peclet numbers suggest that the one-temperature model may also be used in the case of local thermal nonequilibrium. Great attention was paid to the experimental estimation of a high number of parameters (dispersion coefficients, a velocity and six thermocouple positions): their estimated values do not only serve in a mathematical curve fitting exercise but also have to yield physically reasonable and intrinsic values. Three different experimental geometries could confirm the results for k_z.

Nield (1991) has stated that the medium's thermal conductivity can be taken as the geometric average of the thermal conductivities of fluid and solid which falls between those of the serial and parallel approaches, provided thermal conductivities of both phase are not too different from each other.

Yang and Nakayama (2010) have utilized a volume averaging theory to evaluate both stagnant thermal conductivity and thermal dispersion conductivity within porous media. For the stagnant thermal conductivity, a general unit cell model, consisting of rectangular solids with connecting arms in an in-line arrangement, was proposed to describe most homogeneous porous media. The resulting expression for the stagnant thermal conductivity has been validated by comparing the present results with available experimental and theoretical data for packed beds, porous foams and wire screens. As for the thermal dispersion conductivity, a general expression has been derived with help of the two energy equations for solid and fluid phases. It has been revealed that the interfacial heat transfer at the local non-thermal equilibrium controls the spatial distribution of the macroscopic temperature and thus the thermal dispersion activities. The resulting expressions for the longitudinal and transverse thermal dispersion conductivities agree well with available experimental data and empirical correlations.

Experimental Setup

A schematic diagram of the experimental setup is shown in Figure 1. It is made up of a test section (porous medium) which is a stainless steel (AISI 304) pipe with 51.4 mm of inner diameter, 5 mm of wall thickness and 305 mm of length, 2 polyethylene pipes mounted at the each side of the test section where pressure values are collected via gauges and sensors installed in the taps drilled in them, 2 conical reduction fittings to reduce to 32 mm diameter, 2 concentric heat exchangers to absorb the heat applied on the outer surface of the test section, Keithley 2700 data acquisition system and a computer to analyze the collected data, an oscillation generator which consists of is a double-acting cylinder connected to an electrically driven moto-reductor by means of a flywheel and a crank-arm and some other components.

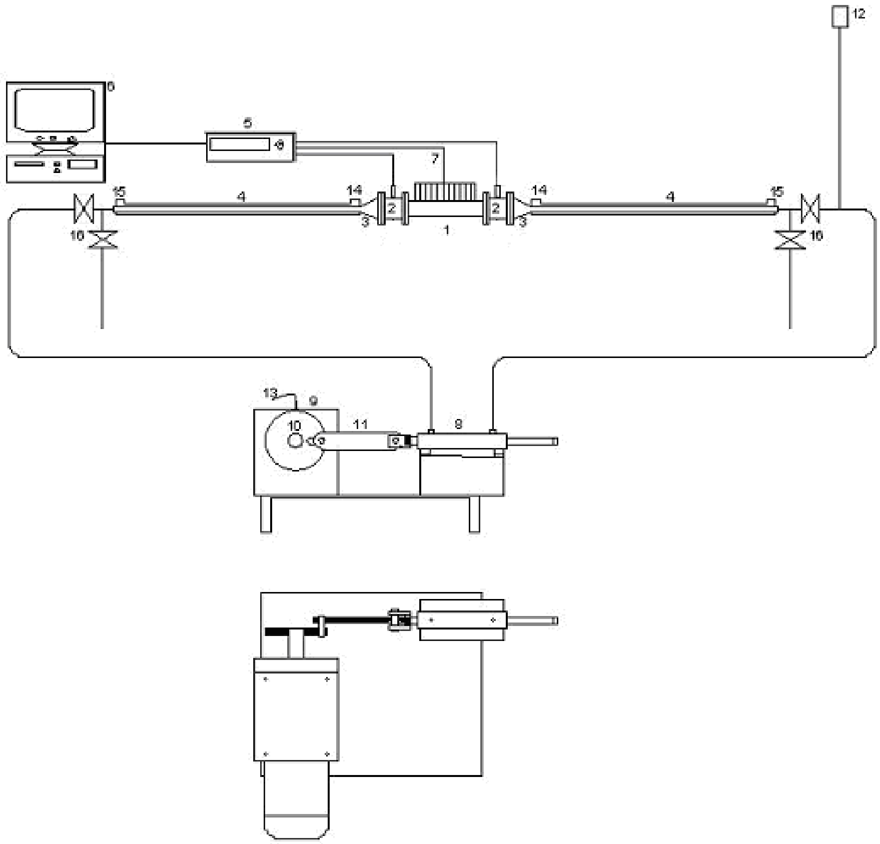

Figure 1 : Experimental Setup :1. Test Section (Porous Medium), 2. PE Pipe, 3. Reduction 4. Concentric Heat Exchanger 5. Keithley 2700, 6. PC 7. Signal Wiring 8. Oscillation Generator, 9. Motoreductor, 10. Flywheel, 11. Crank Arm, 12. Air Purger 13. Inductive Proximity Sensor 14. and 15. Connections to Circulator, 16. Separating Valves.

A custom-made temperature probe shown in Figure 2 is installed at the center of the test section. This probe contains three thermocouples to measure the temperatures at the channel axis (*D/2*) and halfway between the axis and the channel wall (*D/4*). All thermocouples are K-type (Ni-Cr/Ni-Al). Thermocouples attached on the surface and embedded in the pipe wall are utilized to estimate the wall temperature at the water side using Fourier law. Temperature data collected have been time averaged over the number of cycles each set of experiment was conducted. Thus, all the calculations have been performed using these time averaged temperature values.

A more detailed explanation of this experimental setup that also contains in-depth information regarding thermocouple allocations, energy supplying method and heat insulation can be found in the study of Pamuk and Özdemir. Heat inputs throughout this study are shown to be net amounts.

Water is used in this study as the working fluid unlike the majority of the experimental studies in literature. The compressibility of water is considerably affected due to air content, thus the flow rates of half cycles are different which drastically effects the symmetry of heat transfer. In order to overcome this problem a procedure to remove the air from the system is explained in detail in the study of Pamuk and Özdemir.

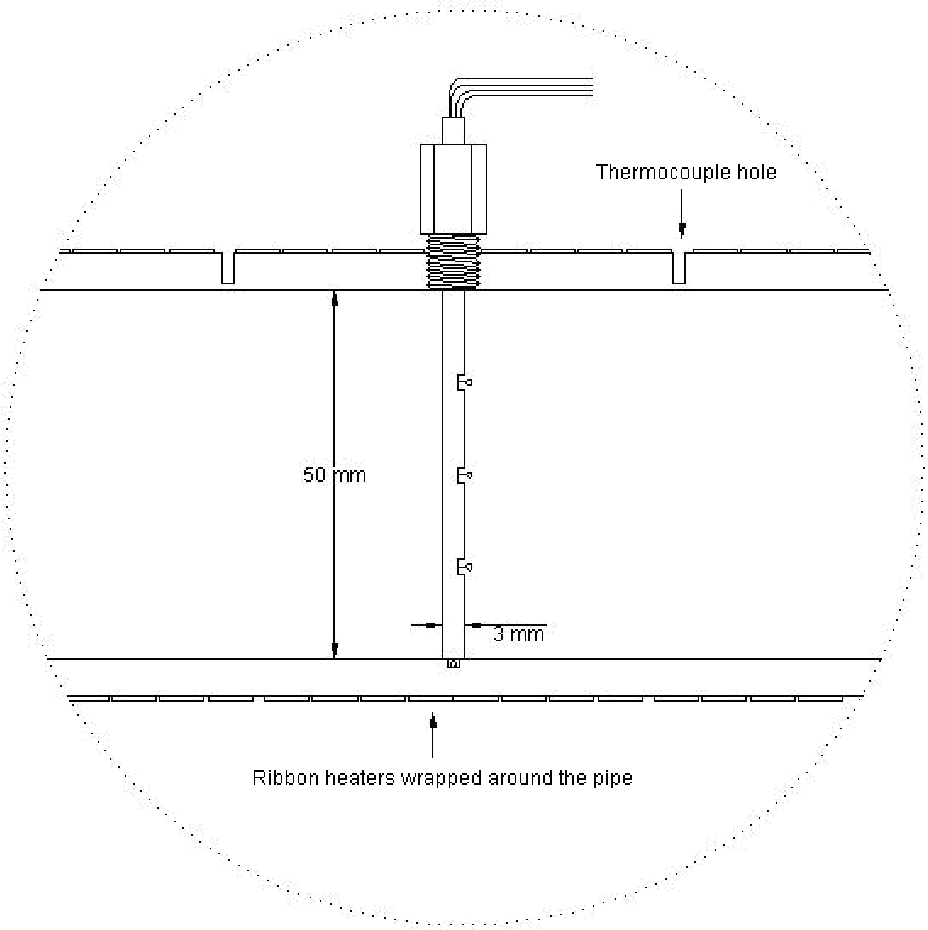

Figure 2 : Temperature probe.

Uncertainty analysis

Uncertainty in the experimental data is considered by identifying the main sources of errors in the primary measurements such as power supplied, dimensions of the test chamber and balls, volume, time and frequency. Then, an uncertainty analysis based on the method described by Figliola and Beasley (2006) is performed. The uncertainties of dimensions, power, temperature, and effective thermal conductivity are estimated to be 1.95%, 1.90%, 2.35%, and 3.58% respectively.

Results and Discussion

Heat transfer experiments for estimating the thermal dispersion conductivity in a porous medium subjected to oscillating flow have been conducted at three different power inputs q, by changing piston stroke, x_{pmax} and frequency output of A/C drive n for both porous media. This way 27 sets of experiments have been conducted. Rotational speed of moto-reductor at 50 Hz line frequency is nominally 70 rpm. Thus rotational speed and corresponding frequency are calculated by $N=(70/50)n$ and $f=N/60$ respectively for a given drive frequency n. Angular frequency is $\omega = 2\pi f$. Actual rotational speed of the moto-reductor determined is to be 69.9 rpm using fft. The displacement of the piston is taken as zero at the rear position inside the cylinder and it becomes a maximum which is equal to the diameter of the flywheel at the forward position. The piston displacement will be equal to the fluid displacement due to the fact that the fluid is incompressible. Hence, at the entrance of the porous medium, the fluid displacement *xm* varies according to

$$x_m(t) = \frac{x_{\max}}{2}(1 - Cos\,\omega t) \qquad (1)$$

where $x_{\max} = x_{p\max} A_p / A = 2RA_p / A$. Here, *R*, A_p and *A* are flywheel radius, cross sectional areas of double acting cylinder and test chamber, respectively. The cross-sectional mean fluid velocity in the channel is

$$u_m(t) = u_{\max} Sin\,\omega t \qquad (2)$$

where $u_{\max} = \omega x_{\max} / 2$.

An inportant issue regarding the temperatures is whether solid and fluid temperatures can be taken equal which denotes the condition "Local thermal equilibrium (LTE)". The criterion set forth by Byun et al. [10] has been applied to find it out. It was found that for both medium, the value $t_p / t_o << 1$ for all conditions that guarantees that LTE exists. Thus, thermocouples at any given location read an average temperature of solid and fluid which are practically equal.
Temporal energy balance on the heated surface at time t can be written as follows:

$$\frac{q}{\pi DL} = q'' = (k_o + k_d)\frac{\partial T(t, r_o, z)}{\partial r} = h\left[T_w(t,z) - T_m(t,z)\right] \qquad (3)$$

Here, dispersion coefficient and temporal heat convection coefficient are dependent on mean flow velocity. On the other hand, mean flow velocity varies harmonically with time. Heat balance on the surface can then be re-written using time averaged temperatures as follows:

$$q'' = (k_o + \bar{k}_d)\frac{\partial \bar{T}(r_o, z)}{\partial r} = \bar{h}(z)\left[\bar{T}_w(z) - \bar{T}_m(z)\right] = \text{constant} \qquad (4)$$

Radial temperature data read out at the center of the porous channel where *z=L/2*, is utilized in estimating the radial temperature gradient $(\Delta T/\Delta r)$ and effective conductivity of the medium (k_e) by utilizing by applying Equation 4 in the form $q'' = -k_e(\Delta T/\Delta r)$. Temperatures used for calculating the radial temperature gradients are measured at wall and half the radius ($r_o/2$). The temperature measured at half the radius (12.85 mm from the wall) is within the channel region which is 5-6 ball diameter wide (17 mm). This facilitates the gradient equation of $\Delta T/\Delta r = \left[\bar{T}_w - \bar{T}_f(r_o/2)\right]/(r_o/2)$ to be used in calculations. Channel region where the highest flow velocity is attained has a higher porosity value than the average porosity of the medium. All the temperature data collected are in the vicinity of 43 ^oC. Therefore, considering Prandtl number is invariant within a small temperature change, kinetic Reynolds number, Re_ω together with non-dimensional fluid displacement, A_o have been used for calculations of effective thermal conductivity in order to account for fluid displacement and oscillation frequency, rather than Peclet number. Temperature data along with other parameters have been given in Table 1. Time averaging for temperature data has been made for 60 cycles. Space averaged Nusselt numbers are within the margin of 66-140. All these details and the way the experiments conducted have been explained in detail in the study of Pamuk and Özdemir.

Table 1 : Temperature data and other parameters.

n, Hz	*L=mm*	A_o	Re_ω	*q", W/m²*	T_w	$T_f(r_o/2)$	$\Delta T/\Delta r$	k_e	k_e/k_f
5	130	1,413	1937	6.091	45,82	41,02	383,8	15,87	25,89
5	130	1,413	1937	8.122	51,99	42,81	734,5	11,06	18,04
5	130	1,413	1937	10.152	61,69	49,94	939,9	10,80	17,62
10	130	1,413	3873	6.091	43,15	37,69	437,0	13,94	22,74
10	130	1,413	3873	8.122	48,18	40,46	617,6	13,15	21,45
10	130	1,413	3873	10.152	53,18	45,04	651,5	15,58	25,42

15	130	1,413	5810	6.091	41,53	37,38	332,3	18,33	29,90
15	130	1,413	5810	8.122	45,17	39,74	435,0	18,67	30,46
15	130	1,413	5810	10.152	52,19	44,22	637,7	15,92	25,97
5	170	1,848	1937	6.091	45,10	39,46	451,3	13,50	22,02
5	170	1,848	1937	8.122	48,55	42,33	497,0	16,34	26,66
5	170	1,848	1937	10.152	54,64	46,36	662,3	15,33	25,01
10	170	1,848	3873	6.091	40,94	36,41	362,6	16,80	27,41
10	170	1,848	3873	8.122	43,74	38,85	391,8	20,73	33,82
10	170	1,848	3873	10.152	46,62	41,06	445,2	22,80	37,20
15	170	1,848	5810	6.091	38,98	35,75	258,4	23,57	38,45
15	170	1,848	5810	8.122	42,07	37,79	342,8	23,69	38,65
15	170	1,848	5810	10.152	45,45	39,57	469,8	21,61	35,25
5	195	2,119	1937	6.091	44,40	37,96	515,4	11,82	19,28
5	195	2,119	1937	8.122	47,12	39,90	576,9	14,08	22,97
5	195	2,119	1937	10.152	51,57	43,66	632,3	16,06	26,19
10	195	2,119	3873	6.091	37,54	34,37	253,6	24,02	39,19
10	195	2,119	3873	8.122	42,23	36,70	442,4	18,36	29,95
10	195	2,119	3873	10.152	46,22	38,49	618,2	16,42	26,79
15	195	2,119	5810	6.091	38,41	33,92	359,4	16,95	27,65
15	195	2,119	5810	8.122	40,92	36,04	390,7	20,79	33,91
15	195	2,119	5810	10.152	43,85	37,30	524,4	19,36	31,58

Figure 3 shows the variation of k_e/k_f with respect to A_oRe_ω, employing the equation $q'' = -k_e(\Delta T/\Delta r)$ where $k_e=k_d+k_o$. Using a linear curve fitting procedure, one obtains a correlation of k_e/k_f with A_oRe_ω. This correlation is in the form of $k_e/k_f= k_o/k_f +C\ (A_oRe_\omega)$ as defined by Yang and Nakayama where Peclet number is employed alternatively. By looking at the correlation carefully, it can be seen that k_o/k_f=18.072, or in other words k_o=11.078 W/m-^{o}C for no flow condition (Eq. 5). This value is very close to paralel model (~10.5 W/m-^{o}C) as given in Nield and Bejan. Therefore it is concluded that parallel model can be employed for estimating the stagnant thermal conductivity (k_o) of the porous medium under oscillating flow. Similarly, the second term in the correlation is $k_d/k_f=0.00145(A_oRe_\omega)$. Aside from a linear correlation, a non-linear curve fitting has been employed to see if data can be better represented (Eq. 6).

$$k_e/k_f = 18.072 + 0.00145(Re_\omega A_o) \tag{5}$$

$$k_e/k_f = -43.7 + 22.322(Re_\omega A_o)^{0.134} \tag{6}$$

However, as can be seen in Fig. 3, both linear and non-linear correlations has the same output for the margin experiments have been conducted. Therefore, for practical purposes, linear model has been adopted as the primary correlation for estimating the effective thermal conductivity in a porous medium of stainless steel balls of 3 mm in diameter under oscillating flow where flow medium is water. Both correlations are valid within the values shown in Table 1.

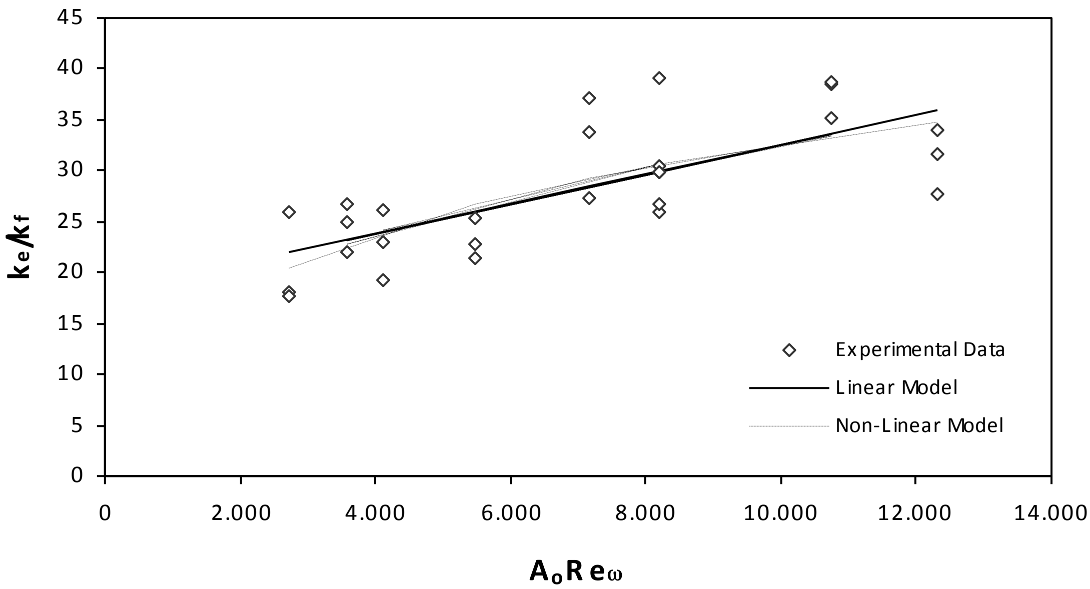

Figure 3 : Variation of effective thermal conductivity vs. A_oRe_ω.

Conclusion

In this experimental study, heat transfer in oscillating flow through porous medium of 3 mm steel balls has been studied. It is shown that the stagnant and dispersion thermal conductivities, thus the effective thermal conductivity of a porous medium of stainless steel balls 3 mm in diameter subjected to oscillating flow can be shown as a linear correlation of A_oRe_ω in the form of $k_e/k_f= C\ (A_oRe_\omega\ )+k_o/k_f$.

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Nomenclature

A	cross-sectional area of the test chamber
A_p	cross-sectional area of double acting cylinder
A_o	non-dimensional displacement defined as $A_o = x_{max}/D$
d	ball diameter
D	inner diameter of the test chamber
Da	Darcy number (K/D^2)
h	heat transfer coefficient
K	permeability
k	conduction coefficient
k_d	dispersion conduction coefficient
k_e	effective conduction coefficient
L	length of the porous medium
n	frequency output of A/C drive (1/s, Hz)
N	rotational speed of moto-reductor (rpm)
$Nu(z)$	local Nusselt number
Nu_L	space-cycle averaged Nusselt number
q	joule heating obtained from ribbon heaters (*IV* Watt)
q''	heat flux at the wall (W/m^2)
r	radial distance from the centerline the test chamber
r_o	radius of the test chamber (*D/2*)

R	radius of flywheel
Re_d	particule based Reynolds Number ($\rho u_m d/\mu$)
Re_ω	kinetic Reynolds Number ($\rho \omega D^2/\mu$)
t	time
t_o	time scale of the variation of the flow or thermal boundary condition
t_p	time scale concerning the thermal inertia of the porous media
$T(t,r,z)$	temperature at radial location r, axial location z at time t
u_m	cross-sectional mean fluid velocity
u_{max}	amplitude of mean fluid velocity
x_p	maximum displacement of the piston (stroke)
x_m	temporal fluid displacement at the inlet of the test chamber
x_{max}	maximum fluid displacement at the inlet of the test chamber

Greek Symbols:

α	heat diffusion coefficient
ρ	density
ω	angular frequency
ν	kinematic viscosity
μ	dynamic viscosity

Subscripts:

f	fluid
m	medium
max	maximum
p	piston
s	solid
w	wall
z	longitudinal direction

References

Byun, S.Y., Ro, S.T., Shin , J.Y., Son, Y.S., Lee, D.-Y. (2006). Transient thermal behavior of porous media under oscillating flow condition, International Journal of Heat and Mass Transfer 49, 5081–5085.

Figliola, Richard S., Beasley, Donald E. (2006). Theory and Design for Mechanical Measurements, John Wiley.

Hsieh, W.H., Lu, S.F. (2000). Heat-transfer analysis and thermal dispersion in thermally-developing region of a sintered porous metal channel, International Journal of Heat and Mass Transfer 43, 3001-3011.

Kaviany, Massoud (1995). Principles of Heat Transfer in Porous Media, Springer.

Kuznetsov, A.V. (2000). Investigation of the effect of transverse thermal dispersion on forced convection in porous media, Acta Mechanica 145, 35-43.

Leong, K.C., Jin, L.W. (2005). An Experimental Study of Heat Transfer in Oscillating Flow Through a Channel Filled Fith Aluminum Foam, International Journal of Heat and Mass Transfer 48, 243-253.

Metzger, T., Didierjean, S., Maillet, D. (2004). Optimal experimental estimation of thermal dispersion coefficients in porous media, International Journal of Heat and Mass Transfer 47, 3341–3353

Nield, Donald A., Bejan, Adrian (2006). Convection in Porous Media, ISBN-3-540-97651-5- Springer-Verlag.

Nield, D. A. (1991). Estimation of the stagnant thermal conductivity of saturated porous media, International Journal of Heat and Mass Transfer Volume 34, Issue 6, June 1991, Pages 1575-1576.

Özgümüş, T., Mobedi1, M., Özkol, Ü., Nakayama, A. (2011). Thermal Dispersion in Porous Media – A Review on Approaches in Experimental Studies, 6th International Advanced Technologies Symposium (IATS'11), 16-18 May 2011, Elazığ, Turkey.

Pamuk, M.T., Özdemir, M. (2012). Heat Transfer in Porous Media of Steel Balls Under Oscillating Flow, Experimental Thermal and Fluid Science, Elsevier (doi:10.1016/j.expthermflusci.2012.04.015).

Vafai, Kambiz (2005). Handbook of Porous Media, Second Edition, CRC Press Taylor & Francis Group.

Yang , C., Nakayama, A. (2010). A synthesis of tortuosity and dispersion in effective thermal conductivity of porous media, International Journal of Heat and Mass Transfer 53, 3222–3230.

Zhao, T.S., Cheng, P. (1998). Heat Transfer In Oscillatory Flows, Annual Review of Heat Transfer, Volume IX.